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The natural frequency of a truss with double braces

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Abstract:

The object of study is a statically determinate plane regular truss with parallel belts. The derivation of the formula for the dependence of the first natural oscillation frequency of the truss on the number of panels and the rigidity of one of the support links is given. **Method.** The forces in the rods are determined in symbolic form by cutting out nodes from the solution of a system of linear equations in the Maple computer mathematics system. The system of equations includes both the forces in the rods and the reactions of the supports. The masses concentrated in the truss nodes have two degrees of freedom. To determine the rigidity of the structure, the Maxwell-Mohr formula is used. Based on a series of separate analytical solutions for trusses with a different number of panels, a general solution is obtained by induction, which is valid for any number of panels. Analytical transformations and numerical solution of the spectrum problem are carried out in the Maple symbolic mathematics system. **Results.** Comparison of the found analytical results with the numerical solution of the problem of the spectrum of natural oscillations of a system with a finite number of degrees of freedom shows its high accuracy, which grows with an increase in the number of panels.

1 Introduction

The existing packages of symbolic mathematics based on well-known algorithms make it relatively easy to obtain analytical solutions to problems of building structures [1]–[5]. However, the area of applicability of such solutions is usually small if the final formulas contain only the dimensions of the structure, loads, and material parameters. In problems with regular trusses, it is also important to take into account the number of panels. In addition to universality, such a solution acquires another property - the possibility of a simple calculation of structures with a large number of rods without loss of accuracy, which is characteristic of numerical methods. The generalization of solutions to an arbitrary number of panels is possible by induction. This is how solutions were obtained for the deformations of planar [6]–[9] and three-dimensional [10], [11] trusses, the problem of oscillation of trusses [12], [13]. Another analytical calculation method is associated with the solution in trigonometric series, implemented in the Maple computer mathematics system. This method is not limited to regular constructions but also does not provide simple calculation formulas. For the first time, the problems of existence and calculation of regular rod systems were posed in the works of Hutchinson R.G. and Fleck N.A. [14], [15]. Regular trusses in connection with optimization problems were considered in [16]–[20]. An inductive approach to the derivation of formulas for analytical solutions to problems of truss vibrations was used in [21].

In this paper, the formula for estimating the first natural frequency of a double-braced truss is derived by induction using the Dunkeley method, and the effect of the stiffness of one of the auxiliary supports on this value is studied.

2 Materials and Methods

Truss scheme. The truss lattice consists of double braces forming rigid triangles with pillar rods, top, and bottom panels (Fig. 1). In addition to the supports common for beam trusses, the truss rests on additional horizontal side support, by changing the rigidity of which it is planned to influence the first (lower) natural frequency of the truss. The truss is statically determinate. Indeed, in a truss with *n* panels, each half-span of length 2a contains $\nu = 8n + 2$ connecting nodes (hinges) and $n_r = 16n + 4$ rods, including four supporting ones. The truss mass is distributed equally among all truss nodes, the rods are assumed to be weightless. Assuming that each node endowed with mass has two degrees of freedom, the number of degrees of freedom of the truss model is $K = 2\nu$. The rigidity of the horizontal lateral elastic connection is selected to change the vibration frequency of the structure. The standard truss calculation scheme, according to which the reactions of the supports are first determined (here there are four of them), and then the forces in the rods are found by successively cutting out the nodes, cannot be implemented here. The truss is externally statically indeterminate. To solve the problem, it is necessary to compose the equilibrium equations for all nodes and find a solution to the system of linear equations, which includes the forces in the rods and the reactions of the supports as unknowns. Note also that the section method in this truss applies only to the edge panels.

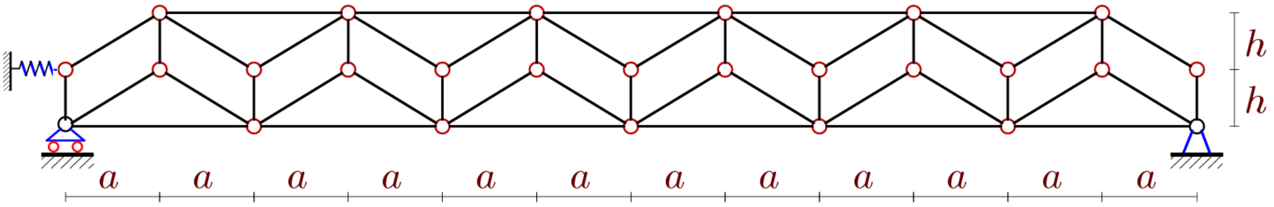

Fig. 1. The truss, n=6

The Maple computer mathematics system allows solving this problem in symbolic form. Using the program [1] and the experience of its application [2-9] in solving similar problems for planar trusses, the coordinates of the nodes are set. The origin of coordinates is located in the left movable support. The numbering of rods and nodes is given in Fig.2.

$$x_i = x_{i+2n+1} = 2a(i-1), y_i = 0, \; y_{i+2n+1} = h, \; i = 1,..,2n+1,$$
$$x_{i+4n+2} = x_{i+6n+2} = 2ai - a, y_{i+4n+2} = h, \; y_{i+6n+2} = 2h, \; i = 1,..,2n.$$

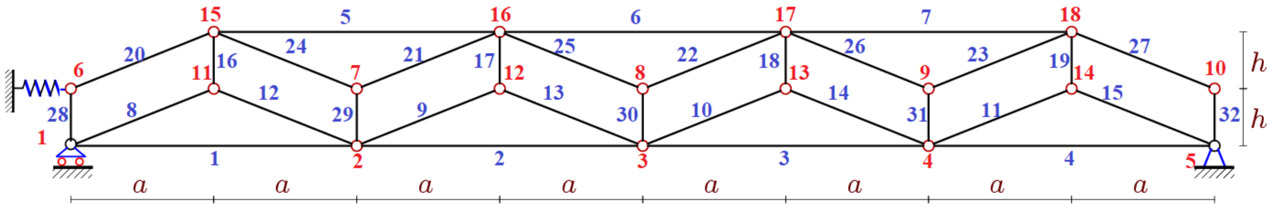

Fig. 2. Numbers of knots and rods at n=2

The structure of the lattice is given by vectors containing the numbers of the ends of the rods. The lower belt, for example, is encoded by lists $N_i = [i, i+1], i = 1,..,2n.$

The matrix of equilibrium equations consists of the direction cosines of the forces calculated from the coordinates of the ends of the rods and their lengths. In the odd rows of the matrix, the direction cosines of the forces with the horizontal axis are entered, and in the even rows, with the vertical axis.

The equations of load oscillations have the form:

$$\mathbf{J}_K \ddot{\mathbf{U}} + \mathbf{D}_K \mathbf{U} = 0 \qquad (1)$$

where $\mathbf{D}_K$ is the stiffness matrix, $\mathbf{U} = [x_1, x_2, ..., x_\nu, y_1, y_2, ..., y_\nu]$ is the vector of displacements of loads, $\mathbf{J}_N = m\mathbf{I}_N$ is the diagonal matrix of inertia, $\mathbf{I}_N$ is the identity matrix, $\ddot{\mathbf{U}}$ is the vector of accelerations of nodes with masses. The inverse of the stiffness matrix $\mathbf{D}_K$ is the compliance matrix $\mathbf{B}_K$, whose elements are calculated using the Mohr integral:

$$b_{i,j} = \sum_{k=1}^{n_r} S_k^{(i)} S_k^{(j)} l_k / (EF). \quad (2)$$

Here $S_k^{(i)}$ is the force in the rod k from the action of a unit force at node *i*, l_k is the length of the rod *k, E* is the modulus of elasticity of the rod material, *F* is the cross-sectional area of the rods. The stiffnesses of the rods are assumed to be the same, except for the lateral horizontal support rod with stiffness EF/γ. The lengths of all support rods are conditionally taken equal to *h*.

An approximate solution according to the Dunkerley method [22]–[24] for the lower estimate of the first vibration frequency ω_D is expressed in terms of the vibration frequencies of individual weights:

$$\omega_D^{-2} = \sum_{k=1}^{2\nu} \omega_k^{-2} \quad (3)$$

where ω_k is the partial oscillation frequency of the mass *m* located at node *k*. In the case of oscillations of one mass, equation (1) has the form: $m\ddot{u}_k + d_k u_k = 0$, where d_k are the stiffness coefficients of the truss node, u_k is the displacement (horizontal or vertical) of the mass. Hence, the oscillation frequency of one load (partial frequency) with a horizontal displacement has the form: $\omega_k = \sqrt{d_k / m}$. The stiffness coefficient is calculated using the Mohr integral through the compliance coefficient: $\delta_k = 1/d_k = \sum_{j=1}^{n_r} \left(\tilde{S}_j^{(k)}\right)^2 l_j / (EF)$. It is denoted here: $\tilde{S}_j^{(k)}$ — the force in the rod with number *j* from the action of a unit force applied to the node where the mass with number k is located. From (3) follows:

$$\omega_D^{-2} = m\sum_{k=1}^{2\nu} \delta_k = m(\Delta_{x,n} + \Delta_{y,n}). \quad (4)$$

3 Results and Discussion

3.1 Derivation of the formula for frequency

For convenience, the sum is divided into two. The sum of the stiffness coefficients for horizontal and vertical displacement of masses is calculated separately.

General view of the solution of the coefficients $\Delta_{x,n}$ and $\Delta_{y,n}$:

$$\begin{aligned} \Delta_{x,n} &= (C_{x,1}a^3 + C_{x,2}c^3 + C_{x,3}h^3 + C_{x,4}a^2h)/(a^2EF), \\ \Delta_{y,n} &= (C_{y,1}(a^3 + c^3) + C_{y,2}h^3)/(h^2EF). \end{aligned} \quad (5)$$

From the sequential solution of the problem of calculating the coefficients for *n*=1, 2, 3, ..., in the case of horizontal mass fluctuations, it follows::

$$\begin{aligned} \Delta_{x,1} &= (38a^3 + 40c^3 + 65h^3 + 10ha^2(\gamma+1))/(2a^2EF), \\ \Delta_{x,2} &= (284a^3 + 288c^3 + 513h^3 + 36ha^2(\gamma+1))/(4a^2EF), \\ \Delta_{x,3} &= (930a^3 + 936c^3 + 1729h^3 + 78ha^2(\gamma+1))/(6a^2EF), \\ \Delta_{x,4} &= (2168a^3 + 2157c^3 + 4097h^3 + 136ha^2(\gamma+1))/(8a^2EF), \\ \Delta_{x,5} &= (4190a^3 + 4200c^3 + 8001h^3 + 210ha^2(\gamma+1))/(10a^2EF), \ldots \end{aligned}$$

In the case of vertical oscillations of the masses, the expressions are a little simpler. The elastic horizontal support connection does not deform, the stiffness coefficient γ is not included in the solution:

$$\Delta_{y,1} = (26(a^3 + c^3) + 57h^3) / (2h^2 EF),$$
$$\Delta_{y,2} = 27(14(a^3 + c^3) + 53h^3) / (2h^2 EF),$$
$$\Delta_{y,3} = 13(1298(a^3 + c^3) + 7521h^3) / (18h^2 EF),$$
$$\Delta_{y,4} = 17(346(a^3 + c^3) + 2707h^3) / (2h^2 EF),$$
$$\Delta_{y,5} = 7(10222(a^3 + c^3) + 100675h^3) / (10h^2 EF), \ldots$$

Using the **rgf_findrecur** and **rsolve** operators from Maple's special genfunc package, you can get the common members of the found sequences:

$$\begin{aligned} &C_{x,1} = (16n^2 + 4n - 1), \quad C_{x,2} = 4n(4n+1), \\ &C_{x,3} = (64n^3 + 1) / (2n), \ C_{x,4} = (4n+1)(\gamma+1), \\ &C_{y,1} = (16n^2 - 1)(32n^2 + 7) / 45, \\ &C_{y,2} = (4n+1)(512n^4 - 128n^3 - 128n^2 + 242n + 15) / 90. \end{aligned} \quad (6)$$

From here and from (4) and (5) follows the final formula for the lower limit of the first natural oscillation frequency of the truss:

$$\omega_D = \sqrt{\frac{EF}{m((C_{x,1}a^3 + C_{x,2}c^3 + C_{x,3}h^3 + C_{x,4}a^2h) / a^2 + (C_{y,1}(a^3 + c^3) + C_{y,2}h^3) / h^2)}}. \quad (7)$$

3.2 Numerical example

The error of the analytical solution (7) with coefficients (6) can be estimated from a comparison with the solution of the problem of oscillation of a system with the number of degrees of freedom $K = 2\nu$, obtained numerically. To find the eigenvalues of a matrix $\mathbf{B}_N$, the operator **Eigenvalues** from the **LinearAlgebra** package of the Maple system is used. The graph (Fig. 3) shows the dependence curves of the first frequency, obtained numerically as the lowest frequency ω_D of the spectrum and by formula (7). The curves almost coincide. Accepted: $EF = 1.29 \cdot 10^6 kN$, $m =$ 500kg, $a =$ 3m, $h =$ 2m, $\gamma = 1$.

The error of the resulting formula is estimated more precisely by the value of the relative error $\varepsilon = (\omega_1 - \omega_D) / \omega_1$. As the number of panels increases, the error of the analytical solution decreases (Fig. 4).

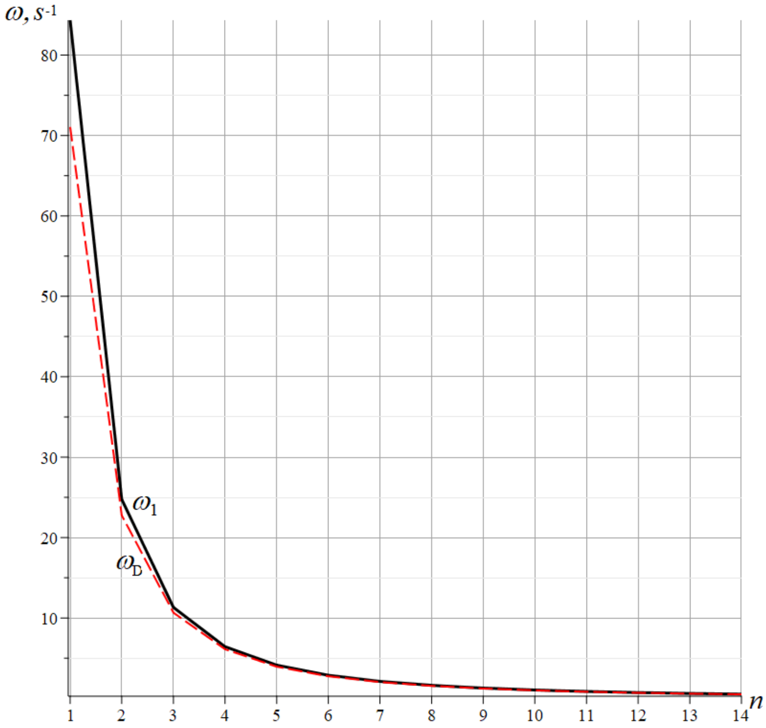

Fig. 3. Frequency dependence on the number of panels

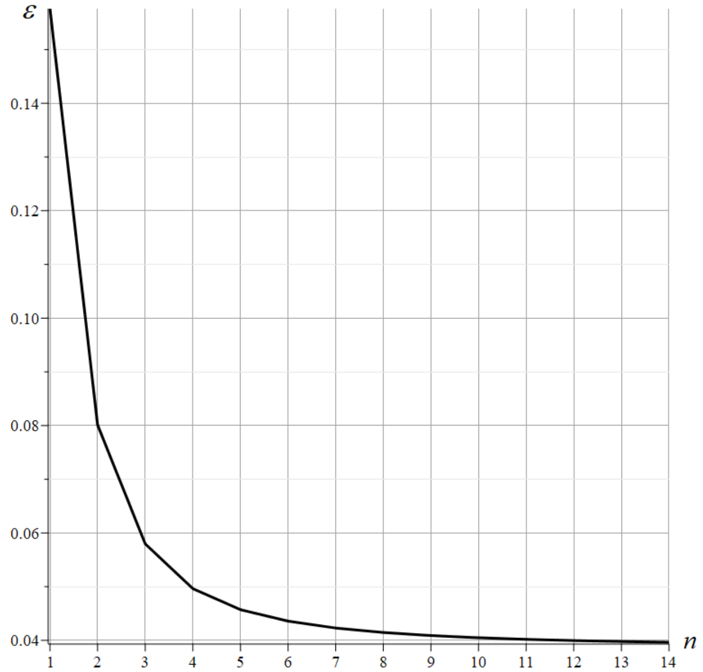

Fig. 4. Dunkerley estimation error depending on the number of panels

The error of the solution, depending on the number of panels, varies from 16%, at n = 1, to 4% with a large number of panels. In the problem of oscillation of a spatial structure, solved by the same method using the Dunkerley estimate [25], [26], the error ranges from 45% to 60%, depending on the number of panels.

The considered scheme of the truss is not spacer. This means that under the action of a vertical load, there is no stress in the left side elastic connection. The entire load is taken by the movable left and fixed right supports. However, the elastic support with rigidity γ affects the horizontal oscillatory movements of the masses. This is manifested in the appearance in (5) of the coefficient γ in the expressions $\Delta_{x,i}$, $i = 1,...,5$. In some practical problems, for example, seismic problems or to avoid resonance, it is necessary to bring the first oscillation frequency out of some dangerous range. Fig. 5

shows how the first natural frequency of the truss changes when it changes depending on the number of panels. For a large number of panels, the effect of horizontal support stiffness is minimal. The nature of this effect echoes the principle of Saint-Venant in the theory of elasticity.

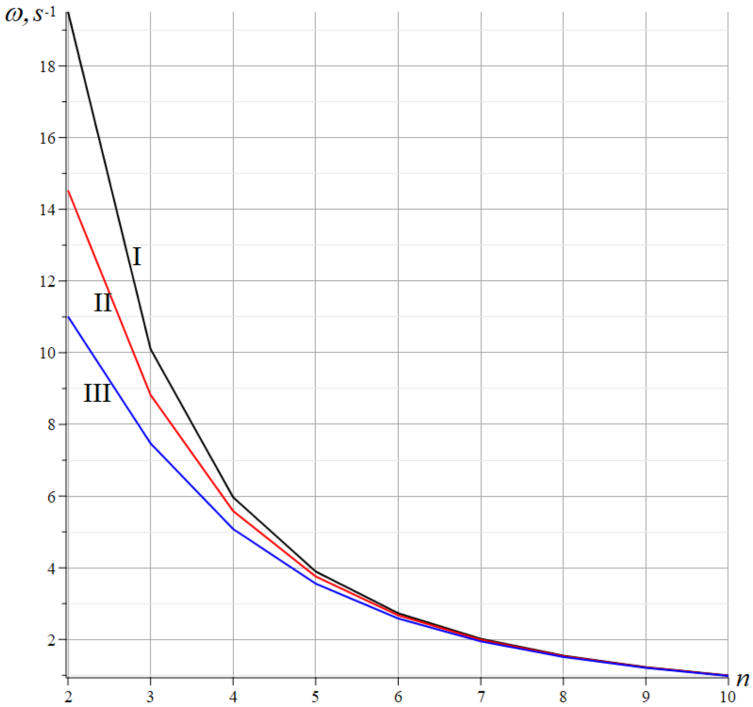

Fig. 5. Influence of the rigidity of the horizontal connection on the oscillation frequency
I — $\gamma = 100$; II — $\gamma = 400$; III — $\gamma = 900$

4 Conclusions

The main results of the work are as follows.

1. Formulas are obtained for calculating the lower limit of the first frequency of natural oscillations.

2. Comparison of the analytical solution with the numerical one shows the high accuracy of the Dunkerley approximation in this problem.

3. The influence of the stiffness of the lateral elastic connection on the oscillation frequency is more pronounced for a small number of panels.

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