Researhe Article
ISSN 2658-5553
Received: December 01, 2022 Accepted: December 19, 2022 Published: December 26, 2022

The module of deformation of a composite material during bending under force and medium load

Bulgakov, Alexey Grigor'evich[1]
Dubrakova, Ksenia Olegovna[1]
Kvasnikova, Anna Nikolaevna[1]
Antoshkin, Vasiliy Dmitrievich[2]
Kotlyarskaya, Irina Leonidovna[3*]

[1] Southwest State University, Kursk, Russian Federation; agi.bulgakov@mail.ru (B.A.G.); dko1988@yandex.ru(D.K.O.); kvasnikova.anna@list.ru(K.A.N.)
[2] National Research Ogarev Mordovia State University, Saransk, Russian Federation; antovd@mail.ru
[3] Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russian Federation; iravassilek@mail.ru
Correspondence:* email iravassilek@mail.ru; contact phone +79095863919

Keywords:

Stability of Timber Systems; Relative deformation; Modulus of deformation of composite material; Strength of timber; Linear creep of timber; Rheological model of deformation of the material; Deformation

Abstract:

The object of research is the module of deformation of a composite material during bending under force and medium load**. Method.** Mathematical modeling of the rheological model. **Results**. The relationship between the determination of the long-term strength of a compressed wood bar and the deformation of structures with the simultaneous manifestation of power and environmental resistance is established. Equations are written for determining the long-term modulus of deformation of wood, considering the influence of importance based on processing the results of the experiment. Using different values measured in bending tests, deflections and deformations associated with the deformation characteristics of the structure material under various stress-strain states, analytical dependencies were obtained that can be used in the calculations of timber structures.

1 Introduction

Various rheological models are used to analyze the long-term strength and stability of wooden structures when assessing their strength resistance. They differ in the degree of accuracy, the number of various factors considered, the level of efficiency when applied for practical purposes, etc. At the same time, there are no design models and experimental data that allow analyzing the deformation of wood structures with the simultaneous manifestation of force and environmental resistance.

Considering a certain difficulty in solving such a problem, it is necessary to create the simplest rheological models of the change in the deformation and strength parameters of wood in time, which make it possible to obtain simple analytical expressions for the criteria of the long-term strength of structural elements made of wood.

The relative deficit of the current value of the studied coefficient of non-equilibrium strength resistance of a tree is described by a certain function that is invariant with respect to all physical and mechanical characteristics of this material: *R, E, 1/s,* etc.

The aim of the work is to formulate an equation for nonlinear creep of tree species at various values of moisture content.

2 Materials and Methods

To construct a criterion for the strength of wood, we use a rheological model of material deformation, consisting of two series-connected elements. The first element (element 0) is described by the used physical deformation model (Fig. 1), the second corresponds to the Kelvin-Voigt model [1, 2]. In accordance with this model, the limiting value of the principal linear deformation wood (or the intensity of shear deformation G*) it's limiting value ε_{ult}:

$$\varepsilon_{ult} = \varepsilon_0 + \varepsilon_1 \tag{1}$$

Where ε_0 is deformations corresponding to element 0 of the used physical deformation model [3] (see Fig. 1), characterizing the process of short-term ("instant") loading, ε_1 is deformations corresponding to the Kelvin-Voigt model [4-9] connected in series with element 0.

When using the presented criterion of wood strength, the determining equation of its long-term strength at $\sigma = const$ is written in the form:

$$\varepsilon \cdot (1 - \sqrt{1 - \frac{\sigma}{R}}) + \frac{\sigma}{E} \cdot (1 - \exp(-\omega \cdot t)) = \varepsilon_{ult} \tag{2}$$

Where the first term corresponds to the approximation of the timber work diagram under short-term loading by a square parabola. Equation (2) makes it possible to find either the long-term strength of wood σ at a given critical time $t = t_{\text{short-term}}$, or directly define $t_{\text{short-term}}$, otherwise $t_{\text{short-term}}$ at a given value σ.

In the event that, under active loading, the stress increases according to an arbitrary law with time $\sigma = \sigma(t)$, the value ε_1 depending on Equation (1) is determined by the expression:

$$\varepsilon_1 = K^{-1} \cdot e^{-\omega t} \cdot \sigma \int_0^t e^{\omega t} dt \tag{3}$$

In (2) and (3) E_1 and K are respectively, the linear modulus of deformation and the modulus of viscous resistance of the Kelvin-Voigt model, $\omega = E_1/K$. Using the above analytical dependences, it is possible to determine the long-term strength of a compressed wood rod. For this, at $\sigma = const$ from equation (3) we find the value ε_1:

$$\varepsilon_1 = \frac{\sigma}{E_1} \cdot (1 - e^{-\omega t}) \tag{4}$$

Since the problem under consideration deals with the long-term strength of wood, instead of stresses σ it is necessary to take σ_{ult}. When approximating the nonlinear dependence of the instantaneous (short-term) deformation by a square parabola, the stress σ is determined from the expression [10]:

$$\varepsilon_0 = \varepsilon_{ult} \cdot (1 - \sqrt{1 - \frac{\sigma_{ult}}{R}}) \tag{5}$$

Where:

$$\varepsilon_{ult} = \frac{2 \cdot R}{E_0}$$

E_0 is initial modulus of elasticity of wood, corresponding point $\sigma=0$. Calculations accepted $E_1=0.75 \cdot E_0$ is the ratio was adopted in the formation of a mathematical model considering the experimental data of E.A. Kvasnikov [11, 12].

Value ω can be found from the analysis of the experimental data of E.A. Kvasnikov, using the following ratio:

$$\omega = \frac{E_0}{K} = \frac{n^*}{n} \cdot (\frac{t}{n})^{n^*-1} \tag{6}$$

Where n is relaxation time; n^* is coefficient characterizing the nonlinearity of viscous resistance. Meaning n and n^* accepted according to experimental data.

3 Results and Discussion

For a quantitative analysis of the long-term strength of a wooden compressed rod, we transform dependence (4) as follows. Let's substitute into this dependence the value $t=\infty$ and value $E_1=0.75 \cdot E_0$. Then, denoting:

$$\varepsilon_{ult} = \frac{2 \cdot R}{E_0}$$

And

$$\varphi_{ult} = \frac{\sigma_{ult}}{R}$$

Then the expression (4) takes the form:

$$\varepsilon_1 = \frac{\varphi_{ult} \cdot \varepsilon_{ult}}{1.5} \quad (7)$$

Substituting (5) and (7) into expression (1), a quadratic equation for φ_{ult} is obtained, and then σ_{ult} will be determined (8,9):

$$\sigma_{ult} = 1 - \frac{R \cdot \varphi_{ult}^2}{2.25} \quad (8)$$

$$\sigma_{ult} = \frac{R \cdot [-1 \mp \sqrt{1 + 1.7(7) \cdot (1 - 0.913^{t^{0.38}})^2}\,]}{0.89 \cdot (1 - 0.913^{t^{0.38}})^2} \quad (9)$$

where σ_{ult} is long-term ultimate strength of wood, *R* is design compression re-sistance of wood, *t* is load application time.

The direct use of the equation of the mechanical state of a material in solving problems of loading and environmental impact is usually not realized due to its cum-bersomeness. For practical use, it is convenient to take the expressions for the long-term deformation modulus [13]:

$$E_{long}(t_0, t) = \frac{\sigma(t)}{\varepsilon(t_0, t)} \quad (10)$$

where $\sigma(t)$ is stresses acting at time *t*; $\varepsilon(t_0,t)$ is relative deformation at the time of observation *t*, set taking into account the effect of the age of the material, its aging properties, the mode and duration of loading.

Assuming that the value of deformations in the area of linear creep of wood at $\sigma = const$ can be determined in accordance with the proposal of Yu.M. Ivanova [14]:

$$\varepsilon(t) = \varepsilon(t_0) \cdot (1 + b \cdot t^{0,21}) \quad (11)$$

where the parameter *b* depends on the moisture content of the wood *(w)* and is defined as:

$$b = \frac{10^{-2}}{0.735 - 0.02086 \cdot w} \quad (12)$$

The formula for determining the long-term modulus of deformation of wood (10), taking into account the influence of importance, can be written as:

$$E_{long}(t_0, t) = \frac{\dfrac{R \cdot [-1 \mp \sqrt{1 + 1.7(7) \cdot (1 - 0.913^{t^{0.38}})^2}\,]}{0.89 \cdot (1 - 0.913^{t^{0.38}})^2}}{\varepsilon(t_0) \cdot (1 + b \cdot t^{0,21})} \quad (13)$$

Calculation of the structure for the second group of limiting states requires knowledge of the value of the elastic modulus (deformation modulus) of the material [15]. The value of the modulus of elasticity can be determined for different types of deformations.

The deflections and deformations measured during bending tests are related to the deformation characteristics of the structure material (elastic and shear modulus – E_x and G_{xy}) by analytical dependences, the accuracy of which is determined by the underlying hypotheses.

The modulus of elasticity of wood can be determined by the value of the deflec-tion from the load during bending tests with a loading scheme with two concentrated loads in the span [16]. The advantages of this method include a close to homogeneous stress state in the area between concentrated forces. Shear deformations occurring in the area of action of the shearing force increase the deflection of the structure, which leads to errors in determining the value of the modulus of elasticity [17]. The influence of shear deformations can be eliminated by measuring the deflection only in the areabetween the concentrated forces (Fig. 1).

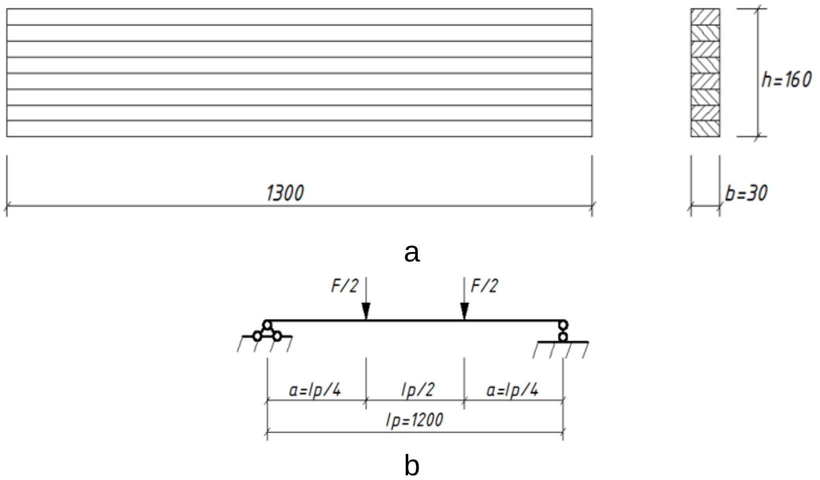

Fig. 1 - The design of the beam (a) and the design scheme (b) of its testing

In the general case of loading a bending element with two concentrated loads, the maximum deflection can be determined using the dependence [18-20]:

$$f = f^*(1+S) \tag{14}$$

where *f** is deflection without regard to shear [17]; *S* is a function that considers the effect of shear on the deflection of the structure [21].

Deflection *f** is determined according to the rules of structural mechanics, which is allowed, considering the linear work of the material of the structure up to destruction.

Function *S*, considering the effect of shifts, can be determined by Mohr's method, built on the principle of virtual work [22-25]. Its value is equal to:

$$S = \mu_0 \cdot \frac{k}{8} \cdot \frac{E_x}{G_{xy}} \cdot \frac{h^2}{l^2} \tag{15}$$

where *k* is coefficient considering the scheme of application of loads; μ_0 is section shape factor (for rectangular section $\mu_0=1.2$).

When loading a beam at two points, the deflection, considering the effect of shear and environmental loading, is determined:

– in the middle of the span:

$$f_c = \frac{F \cdot a \cdot (3l^2 - 4a^2)}{48 E_x J} \left[1 + 2.4 \frac{\left(\dfrac{\dfrac{R \cdot [-1 \mp \sqrt{1 + 1.7(7) \cdot (1 - 0.913^{t^{0.38}})^2}\,]}{0.89 \cdot (1 - 0.913^{t^{0.38}})^2}}{\varepsilon(t_0) \cdot (1 + b \cdot t^{0,21})} \right) \cdot h^2}{G_{xy}(3l^2 - 4a^2)} \right] \tag{16}$$

– at the place of load application:

$$f_M = \frac{F \cdot a^2 (3l - 4a)}{12 E_x J} \left[1 + 0.6 \frac{\left(\dfrac{\dfrac{R \cdot [-1 \mp \sqrt{1 + 1.7(7) \cdot (1 - 0.913^{t^{0.38}})^2}\,]}{0.89 \cdot (1 - 0.913^{t^{0.38}})^2}}{\varepsilon(t_0) \cdot (1 + b \cdot t^{0,21})} \right) \cdot h^2}{G_{xy}(3la - 4a^2)} \right] \quad (17)$$

4 Conclusions

As a result of scientific research, equations of nonlinear creep of wood species at various values of moisture were derived. These equations can be used in further calculations of timber structures.

References

1. Travush, V., Kolchunov, V., Dmitrieva, K. Long-term strength and stability of compressed wood rods. Construction and reconstruction. 2015. 5(61). Pp. 40–46. URL: https://www.elibrary.ru/item.asp?id=24395117 (date of application: 4.07.2022).
2. Travush, V.I., Kolchunov, V.I., Dmitrieva, K.O. Ustojchivost szhatyh sterzhnej iz drevesiny pri odnovremennom proyavlenii silovogo i sredovogo vozdejstviya. Stroitelnaya mekhanika i raschet sooruzhenij. 2016. 2(265). Pp. 50–53. URL: https://www.elibrary.ru/item.asp?id=26700902 (date of application: 4.07.2022).
3. Collection of scientific papers of the 2nd International scientific and practical conference of young scientists, graduate students, masters and bachelors (June 4-5, 2018) / editorial board: Bakaeva N.V. (responsible ed.); Southwest state un-t. Kursk, 2018. Pp 78.
4. S.M. Bazarov, V.I. Patyakin, A.N. Solovyov, A.V. Elkin. Cutting wood material with a string that makes thermoacoustic vibrations. Bulletin of KrasGAU 2011. No. 9. Pp. 282.
5. A. S. Shamaev, V. V. Shumilova, On the spectrum of one-dimensional vibrations in a medium composed of layers of an elastic material and a Kelvin–Voigt viscoelastic material, Zh. Vychisl. math. and mat. Fiz., 2013, Volume 53. No. 2.Pp. 283.
6. Turbin M.V. On the correct formulation of initial-boundary value problems for the generalized Kelvin-Voigt model. Izvestiya Vuzov. Series Mathematics. No. 2006. Pp. 50-58.
7. Zvyagin V.G., Turbin M.V. Mathematical issues of hydrodynamics of viscoelastic media. KRASAND (URSS), 2012. Pp 3-5.
8. Plotnikov P.I., Turbin M.V., Ustyuzhaninova A.S. Existence theorem for a weak solution to an optimal feedback control problem for a modified Kelvin-Voigt model of weakly concentrated aqueous polymer solutions // Dokl. RAN. 2019. V. 488. No. 2. Pp. 133-136.
9. Fedodeev V.I. Rheological models and spatio-time scale effects in the mechanics of dispersive media. VINITI RAN. Moscow region 2008. Pp. 2.
10. Geniev, G.A., Pyatikrestovskij, K.P. Voprosy dlitelnoj i dinamicheskoj prochnosti anizotropnyh konstrukcionnyh materialov. CNIISK im. . Moscow, 2000. Pp. 28.
11. Kvasnikov E.N. Issues of long-term resistance of wood and structural materials made of wood and laminates.1972. - Pp. 95.
12. Damning N.V. Strength of centrally compressed rods from wood under the conditions of force and environmental resistance. Design and construction: Collection of scientific papers. Kursk. 2018. Pp.78.
13. Pyatikrestovsky, K.P., Travush, V.I. Experimental studies on the nature of the vat plywood sheathing as part of spatial structures. Izvestiya Yugo-Zapadnogo gosudarstvennogo Universiteta. 2015. 5(62). Pp. 36–42. URL: https://www.elibrary.ru/item.asp?id=25294569 (date of application: 6.07.2022).

14. Ivanov, Yu.M., Bazhenov, V.A. Studies of the physical properties of wood (elasticity, air permeability, swelling pressure). - M .: Publishing house. Ak. Sciences of the USSR, 1959-75 Pp.
15. Pyatikrestovsky, K.P., Travush, V.I. Nonlinear method programming for calculations of statically indeterminate wooden structures and software systems'' communication to development of improved design standards. Academia. Architecture and construction. 2015. Pp. 115–119. URL: https://www.elibrary.ru/item.asp?id=24073660 (date of application: 6.07.2022).
16. Tarnapolsky Yu.M., Kintsis T.Ya.
Methods for static testing of reinforced plastics. Riga: Zinatne, 1975. Pp. 364.
17. Skuratov S.V. On the determination of the elastic characteristics of construction plywood // Light structures of buildings: interuniversity. Sat. tr. Rostov-on-Don, 1986. Pp. 60-63.
18. Ugolev B.N., Mikhailichenko A.L.The effect of shear force on the value modulus of elasticity of wood during tests for static bending. Woodworking industry. 1962. No. 10.Pp.10-12.
19. Smorchkov A. A. Orlov D. A. Kereb S. A. Kozlov A. V. Smorchkov D. A. Method for determining the elastic modulus in structures from glue-laminated wood. news of the south-western state university. no. 5-2(38). 2011. Pp. 247.
20. Tarnapolsky Yu.M., Kintsis T.Ya. Methods for static testing of reinforced plastics. Riga: Zinatne, 1975. Pp. 364.
21. Ugolev B.N., Mikhailichenko A.L. Influence of transverse force on the value of the modulus of elasticity of wood during tests for static bending // Woodworking industry. 1962. No. 10. Pp.10-12.
22. Aleksandrov, A. V., Travush, V.I., Matveev, A. V. About the calculation of rod structures for stability. Industrial and civil construction. 2002. 3. Pp. 16–20.
23. K.A. Emma, O.V. Makhno, Yu.A. Veselev, G.G. Seferov. Application of the Mohr method for the calculation of three-layer continuous beams. Lightweight structures of buildings: interuniversity. Sat. Rostov-on-Don.1983. Pp. 95-103.
24. Slitskoukhov Yu.V. Registration deflection of glued and plywood beams. Strength and deformability of wood and polymeric materials, compounds elements and structures with their application: Sat. tr. MISI. 1974. Issue. 105. Pp. 122-128.
25. Prokofiev A.S. Glue joint resistance to wooden beams chipping under the existence of the unambiguous pulsating loading. Izvestia universities. Construction and architecture.1973. No. 11. Pp. 19-22.