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The natural frequency of a two-span truss

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Abstract:

The object of the research is a statically definable truss with two spans and a diamond-shaped lattice. One of the supports is a fixed hinge. The other two are movable. The dependence of the first natural vibration frequency of the truss on its size, mass, and also the number of panels is in analytical form. **Methods.** The rigidity of a structure with masses concentrated in its nodes is determined by the Maxwell-Mohr formula. The lower analytical estimate of the first frequency is calculated using the Dunkerley formula. **Results.** The generalization of a series of private solutions for trusses with a sequentially increasing number of panels is made by the induction method. The general terms of the sequence of coefficients are determined from the solution of linear homogeneous recurrent equations. All transformations, including finding the forces in the bars by cutting nodes, are performed in the Maple computer mathematics system. To check the solution, the entire frequency spectrum, including the lowest frequency, is in numerical form. Comparison of the analytical solution with the numerical one shows that the accuracy of the analytical estimate from below is quite high and increases with the number of panels.

1 Introduction

Truss structures are widely used in construction to cover large spans to reduce the consumption of materials used and to lighten structures. The trusses are used as elements of robotic manipulators, in the poles of power lines, in the structures of antennas, street and road signs. Calculation of natural vibration frequencies of such structures is an urgent task, along with the assessment of rigidity and strength. As a rule, calculations are performed numerically in specialized packages based on the finite element method [1]. This makes it possible to obtain solutions to problems for statically indefinite systems and systems with complex boundary conditions to take into account the inelastic or nonlinear properties of the material of the rods. Analytical solutions are possible in the case of simple mathematical models for statically determining trusses [2] - [10]. The reference books [11], [12] collect analytical solutions for flat girder trusses, arches, frames, and consoles. Among the existing models of trusses, one can single out regular structures for which the induction method is applicable [13] - [18]. General questions of the existence of regular statically definable schemes of bar structures were studied in [19], [20]. The deflection of spatial trusses in analytical form using the Maple computer mathematics system [21] was obtained in [22] - [27]. Using the induction method, the dependence of the forces, deflection, and vibration frequencies on the order of the regular structure, for example, on the number of panels or periodic groups of bars, is determined. Such solutions are applicable both for assessing the accuracy of numerical solutions and for preliminary calculations of the designed models, for which the optimal variant can be selected by choosing the order of the regular system.

2 Materials and Methods

The considered truss is a planar beam structure with a diamond-shaped lattice and additional support in the middle of the span (Fig. 1). This makes the construction outwardly statically indeterminate. Calculation of support reactions is possible only in the system of equilibrium equations for all nodes of the truss. Additional support divides the span of the truss into two equal spans of panels in height *h* and length *a*. Despite the four external links (elastic rods of length q), the structure is statically definable. It contains $2n+6$ hinges where $n=2n_0$—the number of rods, including four support rods $\nu=4n+4$. Writing down two equilibrium equations for each node, it is possible to obtain a closed system of equations for the forces in the rods and the reactions of the supports, which is necessary for solving the problem. The theoretical issues of the existence and calculation of regular statically definable farms were studied

Consider a model of a truss, the inertial properties of which are modelled by masses located at the nodes of the lower belt.

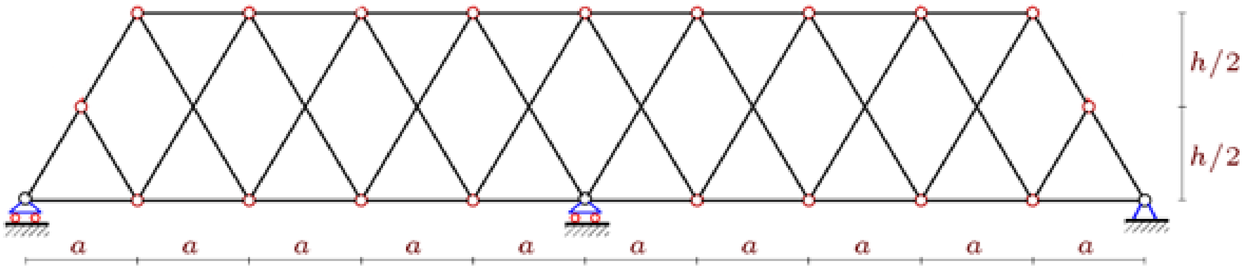

Fig 1. The truss at n=10

Assuming that the masses vibrate only along the vertical y-axis, we find that the number of degrees of freedom of the structure is equal $K=2n_0-1$. In matrix form, the system of equations of motion of the masses is written as follows:

$$m\mathbf{I}_K\ddot{\mathbf{Y}}+\mathbf{D}_K\mathbf{Y}=0. \tag{1}$$

Here $\mathbf{Y}$ is the vector of all vertical displacements of the masses, $\ddot{\mathbf{Y}}$ is the vector of accelerations, $\mathbf{I}_K$ is the unit matrix, $\mathbf{D}_K$ is the stiffness matrix. In the case of harmonic oscillations with frequency ω, the relationship $\ddot{\mathbf{Y}}=-\omega^2\mathbf{Y}$ is valid. The matrix $\mathbf{D}_K$ is the inverse of the compliance matrix $\mathbf{B}_K$. The elements of this matrix are calculated using the Maxwell-Mohr formula:

$$b_{i,j}=\sum_{\alpha=1}^{\nu-4}S_\alpha^{(i)}S_\alpha^{(j)}l_\alpha/(EF)+\sum_{\alpha=\nu-3}^{\nu}S_\alpha^{(i)}S_\alpha^{(j)}l_\alpha/(EF_h). \tag{2}$$

Standard designations have been introduced: $b_{i,j}$ - displacement of the node i from the action of a unit dimensionless force applied to the node j, by force $S_\alpha^{(i)}$ in rods with numbers α from the action of a unified force applied to the node i, where the mass m is located in the direction of movement of the mass, l_α - the length of the rod α. The first sum refers to the bars of the chords and the lattice, for which the same stiffness EF is assumed, the second sum corresponds to four support bars with rigidity $EF_h=EF/r$, where r is the dimensionless coefficient of the relative stiffness of the support bars of length q. It is assumed that the bars are numbered so that the last in the list of bars are the support bars (Fig. 2).

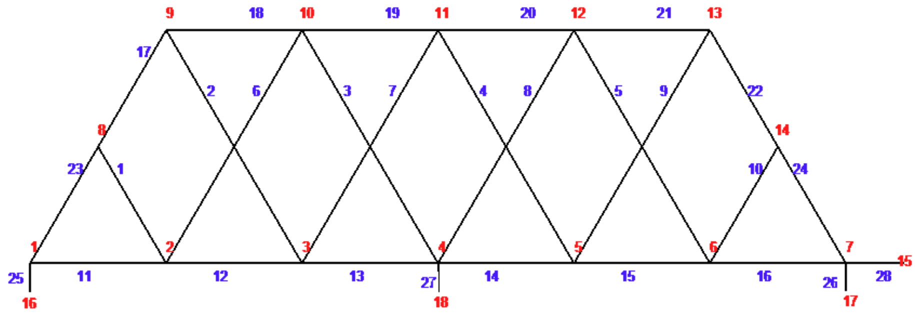

Fig. 2. Numbering of elements and nodes, $n_0 = 3$

Multiplying equality by a matrix $\mathbf{B}_K$, we reduce the problem to the problem of the eigenvalues of a matrix $\mathbf{B}_K$: $\mathbf{B}_K\mathbf{Z} = \lambda\mathbf{Z}$, where $\lambda = 1/(\omega^2 m)$ are the eigenvalues of the matrix $\mathbf{B}_K$. An approximate analytical solution for the lower estimate ω_D of the first frequency ω_1 is sought by the Dunkerley formula [28]–[30]:

$$\omega_D^{-2} = \sum_{p=1}^{K} \omega_p^{-2}. \tag{3}$$

There ω_p are partial frequencies.

Calculation of forces for trusses with a different number of panels shows that the problem has no solution for an even n_0 number of panels. A separate calculation of the determinant of the system of equilibrium equations showed that, in this case, it is equal to zero. The explanation for this effect is related to the kinematic variability of the structure. As an example, consider the case of an admissible pattern of the distribution of the node velocities $n_0 = 2$. The rods 1, 6, 7, 10, 15, and 16 rotate around the bearing hinges. The supporting joints 1, 3, 5, as well as 7 and 9, remain stationary. The rods 3 and 4 move translationally at a velocity u. The rods 2 and 5 are motionless. The rest of the rods instantly rotate. The triangle with vertices 1, 2, and 6 is rigid. Hence, we have the ratio of velocities $u/a = 2u'/c$, where $c = \sqrt{a^2 + h^2}$.

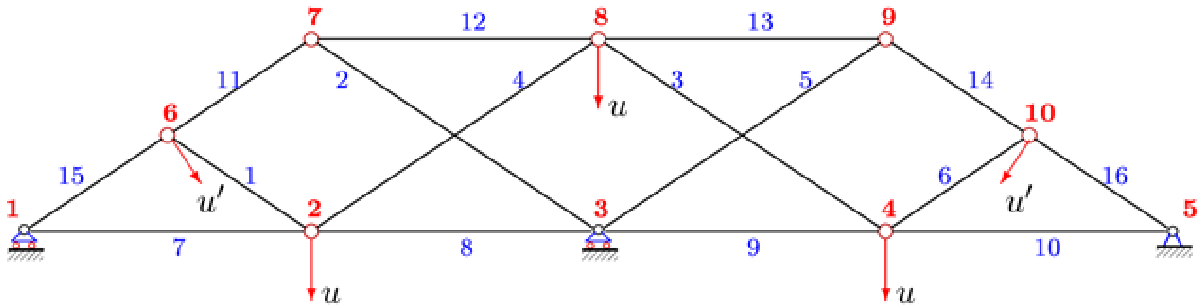

Fig 3. Virtual node velocities, $n_0 = 2$

Next, we take an odd number of panels in half the span $n_0 = 2k + 1$.

3 Results and Discussion

3.1 The lower estimate of the vibration frequency by the Dunkerley method

To calculate the partial frequencies, we compose the equation of motion for a separate mass. Consider the case of rigid support rods, *r* = 0. Assume movement along the *y*-axis:

$$m\ddot{y}_p + D_p y_p = 0, \quad p = 1, 2, ..., K. \tag{4}$$

The stiffness coefficient D_p is inverse to the compliance coefficient, which, like in a system with *K* degrees of freedom, is calculated by the Maxwell-Mohr formula:

$$\delta_p = 1/D_p = \sum_{\alpha=1}^{N-4} \left(S_\alpha^{(p)}\right)^2 l_\alpha/(EF) + \sum_{\alpha=N-3}^{N} \left(S_\alpha^{(p)}\right)^2 /(EF_h) \tag{5}$$

In fact, in such a setting, only the diagonal elements of the matrix $\mathbf{B}_K$ are calculated. When $y_p = A_p \sin(\omega t + \varphi)$ it follows, $\omega_p = \sqrt{D_p/m}$. Then we have:

$$\omega_D^{-2} = m\sum_{p=1}^{K}\delta_p = m(\Delta_k + \Delta'_k). \tag{6}$$

Calculation of a series of trusses with a different number of panels showed that the coefficient Δ_n has a form that does not depend on the parameter *n*:

$$\Delta_1 = (224a^3/9 + 8c^3)/(h^2EF),$$
$$\Delta_2 = (896a^3/5 + 24c^3)/(h^2EF),$$
$$\Delta_3 = (672a^3 + 48c^3)/(h^2EF),$$
$$\Delta_4 = (16352a^3/9 + 80c^3)/(h^2EF),$$
$$\Delta_5 = (4032a^3 + 120c^3)/(h^2EF)...$$

Where $c = \sqrt{a^2 + h^2}$. The property of keeping the shape of the solution takes place for regular constructions, let's write the solution in the form:

$$\Delta_k = (C_1a^3 + C_2c^3)/(h^2EF). \tag{7}$$

To find the common members of the sequences obtained, first, using the Maple system operators, the recurrence equations were found, which they satisfied. In the problem under consideration, it was required to calculate ten farms with a number $k = 1,...,10.$. Note that symbolic conversions in Maple are rather slow. The time for calculating the natural frequencies of each subsequent truss is approximately twice as long as the previous one. Solving recurrent equations gives expressions for determining the coefficients:

$$C_1 = (28k(k+1)(7k^2 + 7k + 6))/45, \quad C_2 = 4k(k+1).$$

Similarly, for the term corresponding to the deflection of the supports, we have

$$\Delta'_k = rq(22k^2 + 16k + 3)/(6(2k+1)EF_h).$$

Finally, we have an analytical estimate for the lower frequency according to Dunkerley:

$$\omega_D^{-2} = m\left((C_1a^3 + C_2c^3)/(h^2EF) + rq(22k^2 + 16k + 3)/(6(2k+1)EF_h)\right). \tag{8}$$

3.2 Numerical solution of the problem of the first frequency of a system with many degrees of freedom

To estimate the accuracy of the analytical solution, let us find the first frequency numerically from the spectrum of frequencies of natural vibrations of the structure using the special operator Eigenvalues in the Maple system, which is used to find the eigenvalues and vectors of the matrix. Consider a truss with dimensions $a = 3\text{m},\ h = 4\text{m},\ q = 1\text{m}.$. The cross-sectional area of the lattice rods and the support rods is the same $F = 10.0\,\text{sm}^2$. Modulus of elasticity of steel $E = 2.1\cdot 10^5$ MPa, masses in nodes: $m = 200\,\text{kg}$. Figure 3 shows the curves of the dependence of the frequency obtained analytically and numerically for $r = 1$.

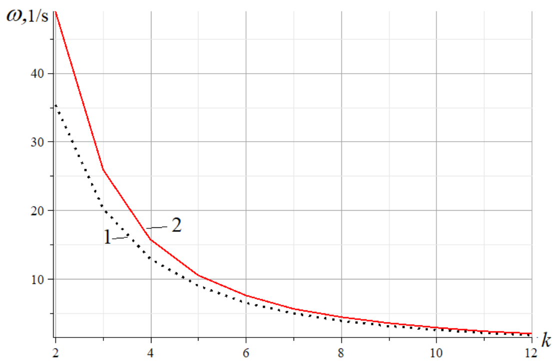

Fig 3. 1 – the first frequency ω_1 and its lower estimate according to Dunkerley at $a = 3\text{m},\ h = 4\text{m},\ q = 1\text{m},\ EF_h = EF$, 2 – is the frequency ω_1 obtained numerically, $r = 1$

As can be seen from the graphs obtained, the analytical evaluation error is small and rapidly decreases with an increase in the number of panels. This makes the obtained analytical solution especially attractive for farms with a large number of panels, where the counting time increases with the number of panels, and the accuracy, due to the inevitable accumulation of rounding errors, decreases.

To clarify the error of the solution, we introduce a relative value $\varepsilon = (\omega_1 - \omega_D)/\omega_1$. Figure 4 shows the change in this value depending on the number of panels. With an accuracy of 9% to 28%, the analytical grade gives a satisfactory result.

Numerical experiments with the selection of the stiffness of the supports have shown that this parameter has a very insignificant effect on the value of the first natural frequency.

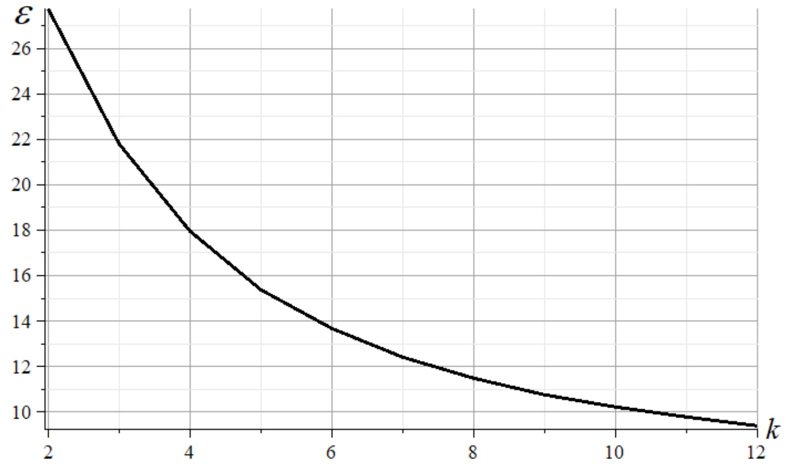

Fig 4. Dunkerley estimate error

4 Conclusions

A mathematical model of a spatial statically definable farm has been built. As a result of the analysis of fluctuations, the following conclusions can be drawn:

1. The Dunkerley score for an arbitrary number of panels is compact and gives acceptable accuracy, especially with a large number of panels.

2. With an increase in the height of the truss, the accuracy of the analytical solution increases.

3. The solution obtained for the rhomboid structure not only describes well the dependence of the frequency on the number of flat truss panels but also gives, in this case, greater accuracy.

5 Acknowledgements

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