alfa build
Research Article
ISSN 2658-5553
Received: July 10, 2021 Accepted: October 19, 2021 Published: October 19, 2021

Deformations of a Triangular Trussed Rafter With an Arbitrary Number of Panels: An Analytical Solution

Kirsanov, Mikhail Nikolaevich[1]

[1] Moscow Power Engineering Institute, Moscow, Russian Federation
Correspondence: email c216@ya.ru; contact phone +79651833534

Keywords:

Trussed Rafter; Analytical Solution; Deflection; Maple, Induction

Abstract:

The object of the study is a planar, statically determinate lattice truss on two supports. The upper belt of the structure has a triangular outline. The task is to obtain formulas for the dependence of the deflection and displacement of the movable support on the number of panels. **Method**. To obtain the deflection value, the Maxwell-Mohr formula is used. The forces in the rods are found by cutting out nodes from the solution of a system of linear equations. All transformations and calculations are performed symbolically in the Maple computer mathematics system. The generalization of a series of particular solutions obtained for trusses with a sequentially increasing number of panels to an arbitrary case is performed by induction using Maple operators. **Results.** Three types of loads are considered, including lateral wind load. The calculated formulas obtained contain the coefficients in the form of polynomials in the number of panels and, in comparison with similar solutions for other structures, have a very compact form. The distribution of forces over the elements of the lattice is given. Formulas are derived for some of the rods that are most critical to the loss of strength or stability.

1 Introduction

The calculation of building structures, including trusses, is traditionally carried out using numerical methods, mainly by the finite element method [1]–[9]. If for a model of an individual structure such an approach is quite sufficient, then for calculating models of the same type, for which the selection of options for their parameters (loads, sizes) is required, it is desirable to have some simple analytical solutions in the form of finite formulas that are convenient for use. The presence of formula even for a simplified design model of the model is a good help for the designer. Especially for calculating regular systems, formulas that include regularity as a function. This makes it possible to evaluate the numerical solutions of high-order systems. General questions of the existence and modeling of statically determinate regular rod systems are considered in [10], [11]. Some solutions for deformations of planar trusses were obtained in [12]–[17]. In these works, for general analytical solutions for an arbitrary number of panels, the induction method and the operators of the Maple computer mathematics system are used [18]. Various algorithms for calculating vibrations of regular rod systems are proposed in [19], [20]. Methods for analyzing the deformation of regular rod systems were studied in [21], [22]. The handbook [23] contains formulas for calculating the deflections of planar regular trusses, arches, and frames. In [24] various options for optimizing regular trusses, including domes, with a large number of design numbers are considered. Algorithms for solving the problem of spline generation of three-dimensional lattice structures (trusses) were studied in [25].

2 Materials and Methods

A diagram of a statically determinate truss with a triangular outline of the upper chord and a diagonal lattice is proposed (Fig. 1). The truss has an odd number of panels along the bottom chord $n = 2k + 1$. The length of each panel is $2a$. The total height of the truss is nh. The length of the rods in the upper chord is $c = \sqrt{a^2 + h^2}$. Let us consider the case of uniform loading of the nodes of the lower chord, the case of loading the upper chord, and the lateral load. A similar truss of triangular shape, but with a rectangular lattice, is designed for deformations in analytical form for an arbitrary number of panels in [26].

To calculate the forces in the rods, we use the program [27] written with the Maple computer mathematics system. The coordinates of the nodes and the order of connecting the rods are entered into the program, in the same way as the edges and vertices of a graph are specified in discrete mathematics.

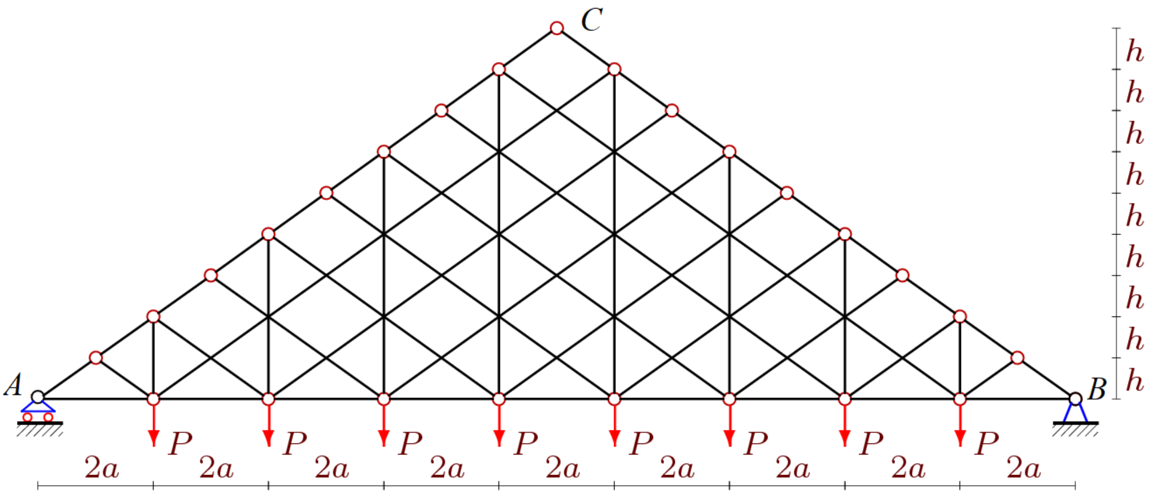

Fig. 1. Truss, *n*=9. The uniform load on the lower chord.

The matrix $\mathbf{G}$ of the system of equations for the equilibrium of the nodes consists of the direction cosines of the forces. The vector on the right side contains the projections of external forces applied to the nodes. The unknowns of this system also include three support reactions. The order of the matrix of equilibrium equations for the nodes of the considered truss is $N = 6n$. To calculate the displacements, the Maxwell-Mohr formula is used. It is assumed that the stiffnesses of the rods are equal and that the supports are non-deformable.

3 Results and Discussion

3.1 Calculation of forces

As a result of the calculation, the program gives the forces in the members for a truss with a specific number of panels in analytical form. Moreover, the formulas for the forces in some rods are cumbersome. It is better to construct a diagram of the distribution of forces based on the results in numerical form (Fig. 2). In the diagram, the red bars are stretched, the blue bars are compressed, and the forces in the black bars are zero. The thickness of the lines is conventionally proportional to the modules of forces. Force values have been rounded. For calculations, dimensions $a = 3\text{m},\ h = 2\text{m}$ were taken. The numerical values of the forces are related to the magnitude of the load *P*. As expected, the most stretched bar S_{I} was found in the middle of the span. The most compressed bar is the lowest bar of the upper chord S_{II}. Direct calculation of the forces in these rods by the method of sections or cutting of nodes for this truss is impossible. The value of the greatest force depending on the number of panels can be obtained by induction based on the results of calculations of trusses with a sequentially increasing number of panels. We have a sequence $S_{\mathrm{I}} / P = 2a/h,\ \ 2a/h,\ \ 4a/h,\ \ 4a/h,\ \ 6a/h, \ldots$. The common term in this sequence is $S_{\mathrm{I}} / P = a(2k+1-(-1)^k)/(2h),\ k = 1,2,\ldots$. Similarly, we have $S_{\mathrm{II}} / P = -ck/h,\ k = 1,2,\ldots$. In the case of a load on the nodes of the upper chord (Fig. 3), the nature of the distribution of forces in the rods will not qualitatively change. Dependencies of forces in critical rods will be as follows: $S_{\mathrm{I}}/P = a(4k+2-(-1)^k)/(2h)$, $S_{II}/P = -c(1+4k)/(2h)$, $k=1,2,\ldots$

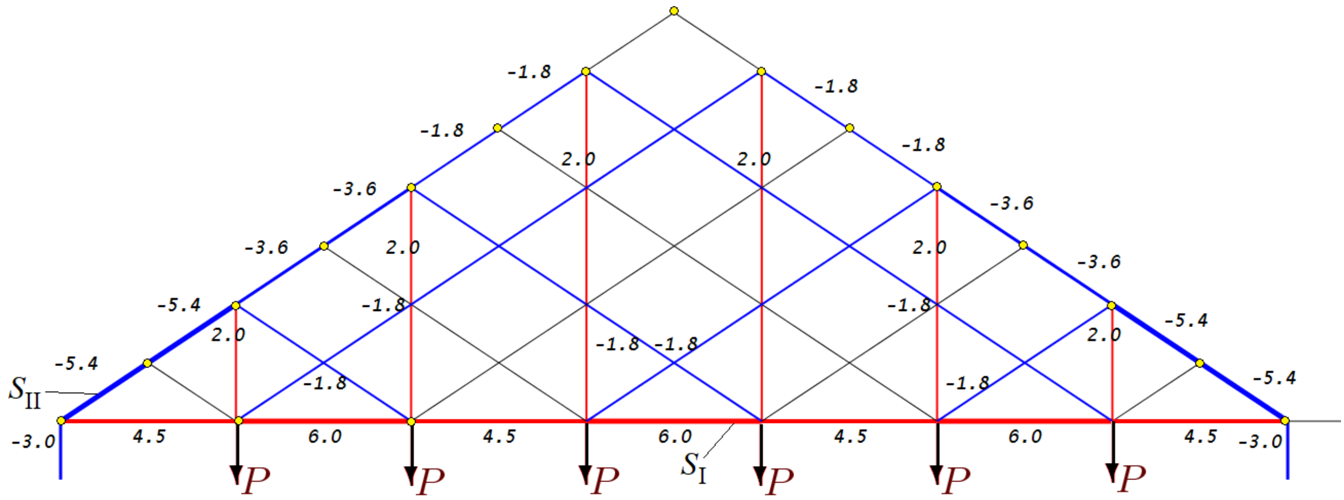

Fig. 2. The distribution of forces in the rods, *n*=7

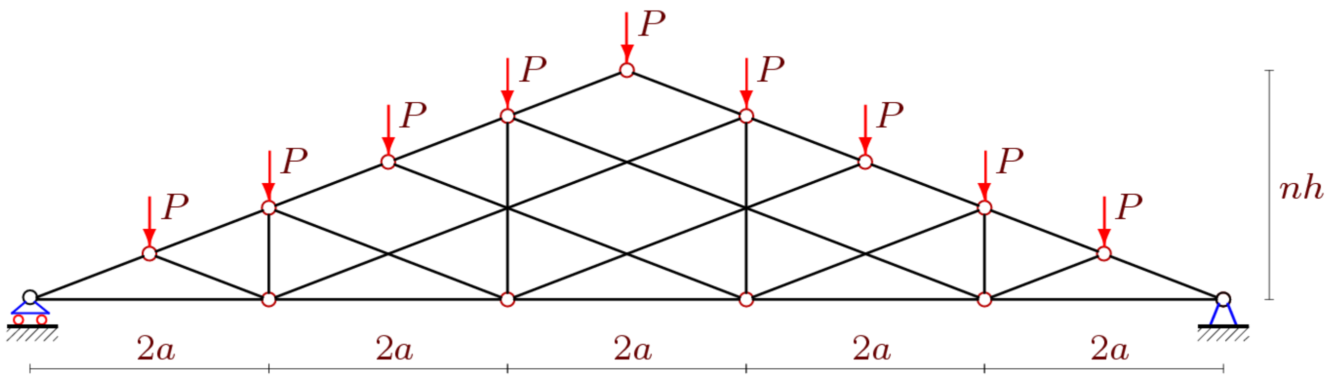

Fig. 3 . Truss, *n=5*. The uniform load on the upper chord.

3.2 Structural deformations

Let us derive the formula for the dependence of the deflection of the structure on the number of panels. The deflection will be fixed by the vertical displacement of the vertex *C*. Assuming that the stiffnesses of the rods EF are the same, the displacement will be calculated using the Maxwell – Mohr formula:

$$\Delta = \sum_{\alpha=1}^{N-3} S_\alpha^{(P)} S_\alpha^{(1)} l_\alpha / (EF). \tag{1}$$

The standard notation is used here: $S_\alpha^{(1)}$ — the force in the rod number α from the action of a single vertical force applied to the point *C*, $S_\alpha^{(P)}$ — force from the action of a distributed load, l_α — the length of the rod. The summation applies to all bars, except for three support bars, which are assumed to be non-deformable. A sequential calculation of the deflection for trusses with an increasing number of panels gives the following results:

$$\begin{aligned}
&k=1, \quad \Delta = 2P(2a^3+c^3)/(h^2EF),\\
&k=2, \quad \Delta = 6P(2a^3+c^3)/(h^2EF),\\
&k=3, \quad \Delta = 12P(2a^3+c^3)/(h^2EF),\\
&k=4, \quad \Delta = 20P(2a^3+c^3)/(h^2EF),\ldots
\end{aligned} \tag{2}$$

To generalize solutions to an arbitrary number of panels, we use the Maple system operators. We get the required formula

$$\Delta = Pk(k+1)(2a^3 + c^3)/(h^2EF). \quad (3)$$

For the case of loading the upper belt, a similar solution has the form

$$\Delta = P((8k^2 + 8k + 1)a^3 + (k+1)(4k+1)c^3)/(2h^2EF). \quad (4)$$

For the frame, arched and pyramidal structures with large dimensions in height, the effect of the horizontal wind load have a noticeable effect on the deformations of the structure and must be taken into account. Consider a load uniformly distributed over the nodes of the lateral side (Fig. 4).

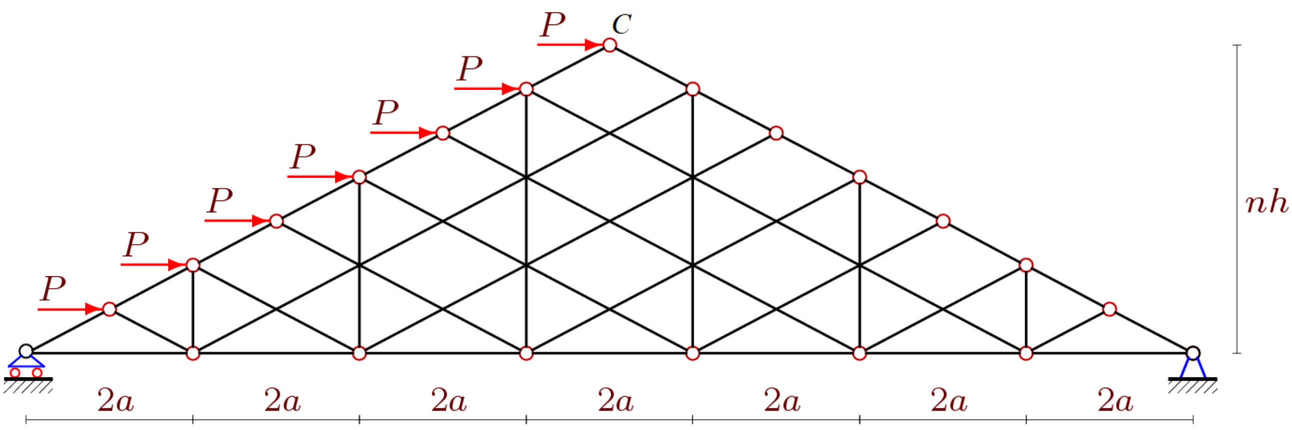

Fig. 4 . Truss, *n*=7. The uniform wind load.

The deflection (vertical displacement of the vertex *C*), calculated by the induction method based on the results of solutions for eight trusses with a sequentially increasing number of panels, has the form

$$\Delta = P(2(4k^2 + 3k + 1)a^3 - kc^3)/(4ahEF). \quad (5)$$

The horizontal displacement of the movable support from the action of the vertical load is manifested to a greater extent in frames and arches [23]. However, even for trusses with a straight bottom chord, the dependence of this value on the number of panels can be useful for the designer. To obtain the deflection value in the Maxwell - Mohr formula (1), the forces $S_\alpha^{(1)}$ must be calculated from the action of a single horizontal force applied to the left movable support. The solution will not be difficult. It will include only the forces in the rods of the lower belt. The forces in the bars of the lattice and the upper chord from the action of a unit force are equal to zero. Omitting simple calculations and the induction process, we give a solution for the case of loading the lower chord: $\delta = 4Pka^2(k+1)/(EFh)$. When loading the upper chord, the dependence of the support displacement on the number of panels will also be quadratic: $\delta = Pa^2(8k^2 + 8k + 1)/(EFh)$. It should be noted that the obtained solutions, in comparison with similar ones known for arches and girder trusses with various lattices [28]–[32], are noticeably simpler in form, although the proposed structure itself is complex.

3.3 Numerical example

Let us give an example of calculating the deflection of a truss for the action of a load along the upper chord. Figure 5 shows the curves of solution (4) for the relative value of the deflection: $\tilde{\Delta} = EF\Delta/(P_sL)$, where $P_s = P(2n-1)$ is the total vertical load on the structure, $L = na = 50\text{m}$ — the span of the truss. Curves plotted for different heights intersect. This means that for trusses with different heights for a certain number of panels, the relative deflection can be the same. The order of alternation of solutions, depending on the height, also changes. For a small number of panels, the greatest deflection is observed for a truss with a height of 1 m, and for a large number of panels, the greatest deflection of the three considered options will be at $h = 3\text{m}$. The analytical form of the solution also shows the existence of asymptotes. The slope of the straight lines follows from the limit $\lim_{k\to\infty}\tilde{\Delta}/k = h/(2L)$. Minima are found on all curves. With increasing truss height, the minimum shifts to the origin. The curves plotted for the case of loading along the lower belt using formula (3) do not qualitatively different from the curves in Fig. 5. The asymptotics of the solutions also coincides. The curves of solution (5) of the problem of the action of wind load look somewhat different (Fig. 6). The solution was obtained if $P_s = nP$. The asymptotics in this problem is quadratic: $\lim_{k\to\infty}\tilde{\Delta}/k^2 = -h^2/(2L^2)$.

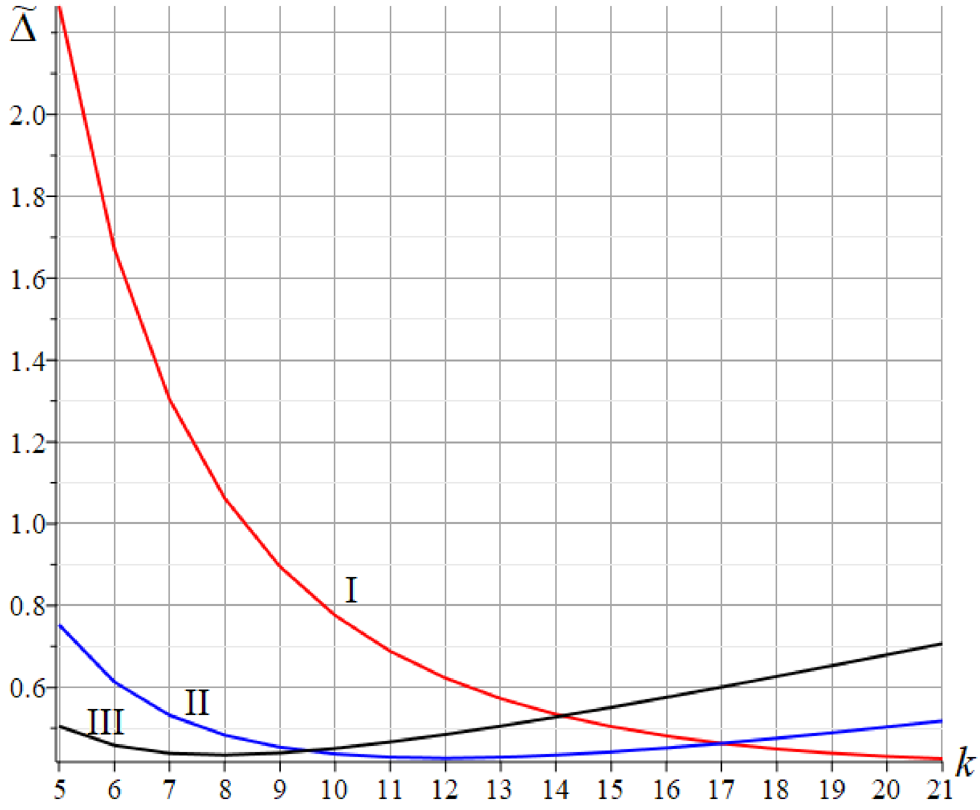

Fig. 5. The dependence of the relative deflection on the action of the load on the upper chord on the number of panels. I — $h = 1\text{m}$; II — $h = 2\text{m}$; III — $h = 3\text{m}$;

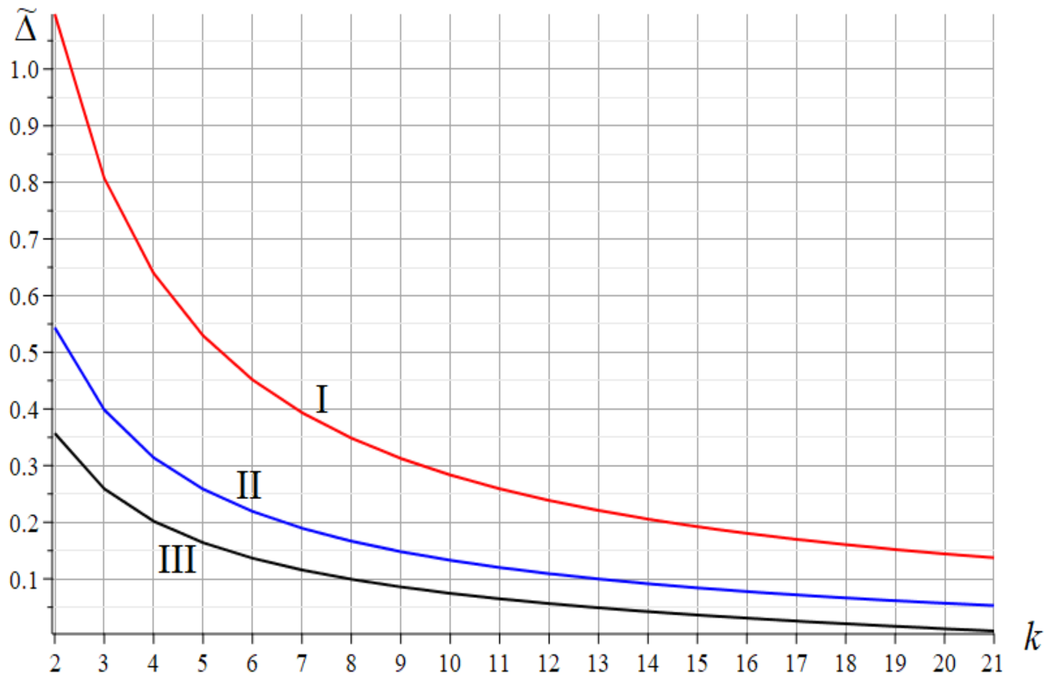

Fig. 6. The dependence of the relative deflection on the action of the wind on the upper chord on the number of panels. I — $h = 1\text{m}$; II — $h = 2\text{m}$; III — $h = 3\text{m}$;

4 Conclusions

The main results of the work are as follows.

1. A statically determinate scheme of a planar truss is proposed.

2. Found the distribution of forces in the rods of the lattice. For the rods that are most dangerous concerning the loss of strength or stability, analytical dependences of the forces on the dimensions of the structure and the number of panels are obtained.

3. Derived formulas for calculating the deflection and displacement of the movable support with three types of loads. The asymptotics of the solutions is found. The existence of a minimum deflection is noted for some types of loads.

5 Acknowledgments

This research has been supported by the Interdisciplinary Scientific and Educational School of Moscow University «Fundamental and Applied Space Research».

References

1. Vatin, N., Havula, J., Martikainen, L., Sinelnikov, A.S., Orlova, A. V., Salamakhin, S. V. Thin-walled cross-sections and their joints: Tests and FEM-modelling. Advanced Materials Research. 2014. 945–949. Pp. 1211–1215. DOI:10.4028/www.scientific.net/AMR.945-949.1211.
2. Khatibinia, M., Naseralavi, S.S. Truss optimization on shape and sizing with frequency constraints based on orthogonal multi-gravitational search algorithm. J. Sound Vib. 2014. 333(24). Pp. 6349–6369.
3. Ye, G., Bi, H., Hu, Y. Compression behaviors of 3D printed pyramidal lattice truss composite structures. Composite Structures. 2020. 233. Pp. 111706. DOI:10.1016/j.compstruct.2019.111706.
4. Abdikarimov, R., Khodzhaev, D., Vatin, N. To Calculation of Rectangular Plates on Periodic Oscillations. MATEC Web of Conferences. 2018. 245. DOI:10.1051/matecconf/201824501003.
5. Khodzhaev, D., Abdikarimov, R., Vatin, N. Nonlinear oscillations of a viscoelastic cylindrical panel with concentrated masses. MATEC Web of Conferences. 2018. 245. DOI:10.1051/matecconf/201824501001.
6. Vatin, N.I., Sinelnikov, A.S. Footway bridges: cold formed steel cross-section. Construction of Unique Buildings and Structures. 2012. 3(3). Pp. 39–51. DOI:10.18720/CUBS.3.5. URL: https://unistroy.spbstu.ru/article/2012.3.5 (date of application: 17.04.2021).
7. Lardeur, P., Arnoult, É., Martini, L., Knopf-Lenoir, C. The Certain Generalized Stresses Method for the static finite element analysis of bar and beam trusses with variability. Finite Elements in Analysis and Design. 2012. 50. Pp. 231–242. DOI:10.1016/j.finel.2011.09.013.
8. Balu, A.S., Rao, B.N. High dimensional model representation based formulations for fuzzy finite element analysis of structures. Finite Elements in Analysis and Design. 2012. 50. Pp. 217–230. DOI:10.1016/j.finel.2011.09.012.
9. Arndt, M., Machado, R.D., Scremin, A. An adaptive generalized finite element method applied to free vibration analysis of straight bars and trusses. Journal of Sound and Vibration. 2010. 329(6). Pp. 659–672. DOI:10.1016/j.jsv.2009.09.036.
10. Hutchinson, R.G., Fleck, N.A. Microarchitectured cellular solids - The hunt for statically determinate periodic trusses. ZAMM Zeitschrift fur Angewandte Mathematik und Mechanik. 2005. 85(9). Pp. 607–617. DOI:10.1002/zamm.200410208.
11. Hutchinson, R.G., Fleck, N.A. The structural performance of the periodic truss. Journal of the Mechanics and Physics of Solids. 2006. 54(4). Pp. 756–782. DOI:10.1016/j.jmps.2005.10.008.
12. Arutyunyan, V.B. Calculation of the deflection of a statically indeterminate beam truss. Postulat. 2018. 6(6). URL: http://vuz.exponenta.ru/1/ar18.pdf.
13. Ilyushin, A.S. The formula for calculating the deflection of a compound externally statically indeterminate frame. Structural mechanics and structures. 2019. 22(3). Pp. 29–38. URL: https://elibrary.ru/item.asp?id=41201106 (date of application: 27.02.2021).
14. Kitaev, S.S. Derivation of the formula for the deflection of a cantilevered truss with a rectangular diagonal grid in the computer mathematics system Maple. Postulat. 2018. 5–1. Pp. 43. URL: http://e-postulat.ru/index.php/Postulat/article/view/1477 (date of application: 3.03.2021).

15. Rakhmatulina A R, Smirnova, A.A. Two-parameter derivation of the formula. Postulat. 2018. 31(5–1). URL: http://vuz.exponenta.ru/1/rsm.pdf (date of application: 1.03.2021).
16. Voropay, R., Domanov, E. Analytical solution of the problem of shifting a movable support of a truss of arch type in the Maple system. Postulat. 2019. 1. URL: http://vuz.exponenta.ru/1/vd.pdf (date of application: 27.02.2021).
17. Rakhmatulina A R, Smirnova, A.A. The formula for the deflection of a truss loaded at half-span by a uniform load. Postulat. 2018. 29(3). URL: http://vuz.exponenta.ru/1/r-s.pdf (date of application: 1.03.2021).
18. Zotos, K. Performance comparison of Maple and Mathematica. Applied Mathematics and Computation. 2007. 188(2). Pp. 1426–1429. DOI:10.1016/j.amc.2006.11.008.
19. Rybakov, L. S., Mishustin, I. V. Small elastic vibrations of planar trusses of orthogonal structure. Mechanics of composite materials and structures. 2003. 9(1). Pp. 42–58. URL: https://elibrary.ru/item.asp?id=11724233 (date of application: 5.07.2021).
20. Ignatiev, V.A. Calculation of regular rod systems. Saratov Higher Military Chemical Military School. Saratov, 1973.
21. Obraztsov, I. F., Rybakov, L. S., Mishustin, I. V. On methods for analyzing deformation of rod elastic systems of a regular structure. Mechanics of composite materials and structures. 1996. 2(2). Pp. 3–14. URL: https://elibrary.ru/item.asp?id=9189956 (date of application: 5.07.2021).
22. Rybakov, L.S. Linear elastic analysis of a spatial orthogonal lattice. Mechanics of composite materials and structures. 2016. 22(4). Pp. 567–584. URL: https://www.elibrary.ru/item.asp?id=28149156 (date of application: 4.07.2021).
23. Kirsanov, M. Trussed Frames and Arches: Schemes and Formulas. Cambridge Scholars Publishing Lady Stephenson Library. Newcastle upon Tyne, GB, 2020.
24. Kazemzadeh Azad, S., Aminbakhsh, S. High-dimensional optimization of large-scale steel truss structures using guided stochastic search. Structures. 2021. 33. Pp. 1439–1456. DOI:10.1016/J.ISTRUC.2021.05.035.
25. Van Sosin, B., Rodin, D., Sliusarenko, H., Bartoň, M., Elber, G. The Construction of Conforming-to-shape Truss Lattice Structures via 3D Sphere Packing. Computer-Aided Design. 2021. 132. Pp. 102962. DOI:10.1016/J.CAD.2020.102962.
26. Kirsanov, M.N., Khromatov, V.Y. Modeling deformations of triangular shape flat truss. Structural Mechanics and Analysis of Constructions. 2017. 275(6). Pp. 24–28. URL: https://www.elibrary.ru/item.asp?id=30638551 (date of application: 8.07.2021).
27. Buka-Vaivade, K., Kirsanov, M.N., Serdjuks, D.O. Calculation of deformations of a cantilever-frame planar truss model with an arbitrary number of panels. Vestnik MGSU. 2020. (4). Pp. 510–517. DOI:10.22227/1997-0935.2020.4.510-517.
28. Petrichenko, E.A. The lower limit of the frequency of natural vibrations of the Fink truss. Structural mechanics and structures. 2020. 26(3). Pp. 21–29. URL: http://vuz.exponenta.ru/pdf/NAUKA/Petr-2020-433.pdf.
29. Vorobev, O.V. On methods of obtaining an analytical solution for the problem of natural frequencies of hinged structures. Structural mechanics and structures. 2020. 24(1). Pp. 25–38. URL: http://vuz.exponenta.ru/pdf/NAUKA/elibrary_42591122_21834695.pdf.
30. Ovsyannikova, V.M. Dependence of deformations of a trapezous truss beam on the number of panels. Structural Mechanics and Structures. 2020. 26(3). Pp. 13–20. URL: https://www.elibrary.ru/item.asp?id=44110286 (date of application: 11.03.2021).
31. Ovsyannikova, V.M. Dependence of the deflection of a planar external statically undeterminable truss on the number of panels. Structural Mechanics and Structures. 2020. 27(4). Pp. 16–25. URL: https://www.elibrary.ru/download/elibrary_44374443_62905709.pdf.
32. Vorobyev, O. About methods of obtaining analytical solution for eigenfrequencies problem of trusses. Structural mechanics and structures. 2020. 1(24). Pp. 25–38. URL: http://vuz.exponenta.ru/PDF/NAUKA/elibrary_42591122_21834695.pdf.