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Estimation of the Natural Vibration Frequency of a Triangular Mast

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Abstract:

The object of research is a spatial statically determine truss with the shape of a triangular pyramid with a cross-shaped lattice. One of the three pillars of the structure is a spherical joint. The other two are a cylindrical joint and a vertical post. In the analytical form, the dependence of the first natural frequency of vibrations of the truss on its size, mass, and number of panels is found. **Method**. The stiffness of a truss with masses concentrated at the nodes is determined by the Maxwell-Mohr formula. The analytical estimate of the first frequency is calculated using the Dunkerley formula. The induction method is used to generalize a series of particular solutions for trusses with a consistently increasing number of panels. The general terms of the sequence of coefficients are determined from the solution of linear homogeneous recurrent equations. All transformations, including finding the forces in the rods by cutting out the nodes, are performed in the Maple computer mathematics system. **Results.** The frequency dependence on the number of panels for trusses with an arbitrary slope of the side faces is found in numerical form. An analytical solution can be obtained for the case of vertical faces of the structure when the truss has the shape of a regular triangular prism. A comparison of the analytical solution with the numerical one shows that the accuracy of the analytical estimate from below increases with an increase in the number of panels.

1 Introduction

Truss structures are widely used in power line supports for mounting antennas, street and road signs. Along with the assessment of the rigidity and strength of such structures, the calculation of the natural frequencies of vibrations is an urgent task. Calculations are usually performed numerically in specialized packages based on the finite element method [1]–[4]. This makes it possible to obtain solutions to problems for statically indeterminate systems, systems with complex boundary conditions, for taking into account inelastic or nonlinear properties of the rod material. In the same cases, when a simple mathematical model of a statically determinate truss is used for the design, an analytical solution is also possible. Among all models of trusses, we can distinguish regular structures. For regular structures, we apply the method of induction to obtain the dependence of the solution on the number of panels or rods. Such solutions are applicable for the evaluation of numerical solutions and for preliminary calculations of the designed models, for which it is possible to choose the optimal option by choosing the order of the regular system. General questions about the existence of statically determinate regular trusses are discussed in [5]–[7]. Formulas for the dependence of the deflection of regular planar trusses by induction in the Maple system are obtained in [8]–[12]. Some analytical calculations of the eigenfrequencies of the plane and spatial trusses are also known [13]–[15]. In [16], [17], the problems of deformations of spatial trusses are solved in an analytical form.

2 Materials and Methods

The model of the mast is a regular triangular pyramid, having at its base three supports at the corners of an equilateral triangle with the side a. Corner *A* has spherical hinge support, modelled by three mutually perpendicular rods. The cylindrical joint in corner *B* structurally consists of two rods. The support at vertex *C* is a vertical post (Fig. 1). The faces of the structure are identical trusses with a cross-shaped grid. The slope of the faces is determined by the ratio of the height of the panel h and the size $b = nr < a/2$ (Fig. 2). At $r = 0$, the faces are vertical, and the truss takes the form of a triangular prism. The truss consists of n panels in height and a top panel of three rods. The total height of the truss is $H = nh + h_1$. The truss contains $N = 9n + 12$ rods, including six rods that support the model. The most common and fairly accurate model of the inertial properties of the truss is used: the mass of the truss is concentrated in all its nodes, except for the three reference ones. Thus, the total mass of the structure is $m(3n+1)$. The frequencies of free vibrations of the structure are determined from the analysis of the system of equations of mass motion. Assuming that the masses move only in the direction of one axis, for example, y, we get that the number of degrees of freedom of the structure is $K = 3n + 1$. In matrix form, the system is written as follows:

$$m\mathbf{I}_K \ddot{\mathbf{Y}} + \mathbf{D}_K \mathbf{Y} = 0. \quad (1)$$

Here $\mathbf{Y}$ is the vector of all mass displacements at the truss nodes, $\ddot{\mathbf{Y}}$ is the acceleration vector, $\mathbf{I}_K$ is the unit matrix and $\mathbf{D}_K$ is the stiffness matrix. In the case of harmonic oscillations with frequency ω, the relation $\ddot{\mathbf{Y}} = -\omega^2 \mathbf{Y}$ is valid. The matrix $\mathbf{D}_K$ is the inverse of the malleability matrix $\mathbf{B}_K$. The elements of this matrix are calculated using the Maxwell-Mohr formula:

$$b_{i,j} = \sum_{\alpha=1}^{N-6} S_\alpha^{(i)} S_\alpha^{(j)} l_\alpha / (EF). \quad (2)$$

Standard designations have been introduced: $b_{i,j}$ — the displacement of the node i from the action of the unit dimensionless force applied to the node j, $S_\alpha^{(i)}$ — the forces in the rods with numbers α from the action of the unit force applied to the node i, in which the mass is located, in the direction of movement of the mass, l_α — the length of the rod α. Multiplying equality (1) by the matrix $\mathbf{B}_K$, we reduce the problem to the problem of the eigenvalues of the matrix $\mathbf{B}_K$: $\mathbf{B}_K \mathbf{Z} = \lambda \mathbf{Z}$, were $\lambda = 1/(\omega^2 m)$ — the eigenvalues of the matrix $\mathbf{B}_K$. An approximate analytical solution for the lower estimate ω_D of the first frequency ω_1 is sought by the Dunkerley formula [18], [19]:

$$\omega_D^{-2} = \sum_{p=1}^{K} \omega_p^{-2}, \quad (3)$$

where ω_p are the partial frequencies.

The values of the force in the rods included in (2) are determined by the method of cutting out the nodes according to the program [20], compiled in the language of symbolic mathematics [21].

3 Results and Discussion

3.1 Analytical solution. The lower estimate by the Dunkerley method

To calculate the partial frequencies, we make up the equation of motion of a single mass. Consider the case of $r = 0$ (a truss in the form of a regular triangular prism). We assume movement on the y-axis

$$m\ddot{y}_p + D_p y_p = 0, \quad p = 1, 2, ..., K. \quad (4)$$

The coefficient of rigidity D_p is the inverse of the coefficient of compliance, which, as in a system with *K* degrees of freedom, is calculated by the Maxwell-Mohr formula

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$$\delta_p = 1/D_p = \sum_{\alpha=1}^{N-6} \left(S_\alpha^{(p)}\right)^2 l_\alpha / (EF). \tag{5}$$

In fact, in this formulation, only the diagonal elements of the matrix $\mathbf{B}_K$ are calculated. From (4) at $y_p = A_p \sin(\omega t + \varphi)$, the solution $\omega_p = \sqrt{D_p / m}$ follows. Taking into account (3), we have

$$\omega_D^{-2} = m \sum_{p=1}^{K} \delta_p = m\Delta_n. \tag{6}$$

The calculation of a series of trusses with a different number of panels showed that the coefficient Δ_n has the form independent of the parameter n:

$$\begin{aligned}
&\Delta_1 = (189a^3 + 270c^3 + 4d^3 + 630h^3)/(54a^2EF),\\
&\Delta_2 = (507a^3 + 2268c^3 + 8d^3 + 5364h^3)/(108a^2EF),\\
&\Delta_3 = (207a^3 + 1152c^3 + 2d^3 + 3906h^3)/(27a^2EF),\\
&\Delta_4 = (957a^3 + 8136c^3 + 8d^3 + 36072h^3)/(108a^2EF),...
\end{aligned} \tag{7}$$

where $c = \sqrt{a^2 + h^2}$, $d = \sqrt{3a^2 + 9h^2}$. The shape-preserving property of the solution holds for regular constructions. At $r \neq 0$, this property is lost, and the analytical solution cannot be obtained. Let's write the solution in the form:

$$\Delta_n = (C_1a^3 + C_2c^3 + C_3h^3 + C_4d^3)/(a^2EF) \tag{8}$$

To find the common terms of the obtained sequences, we first used the operators of the Maple system to determine the recurrent equation that they satisfied.

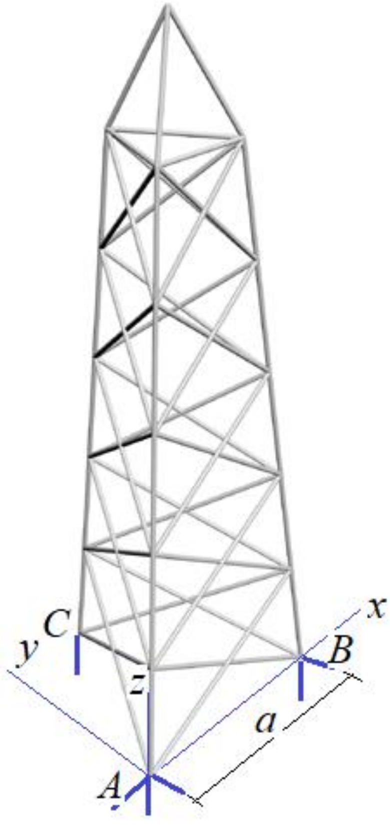

Fig. 1 – Truss, n=5

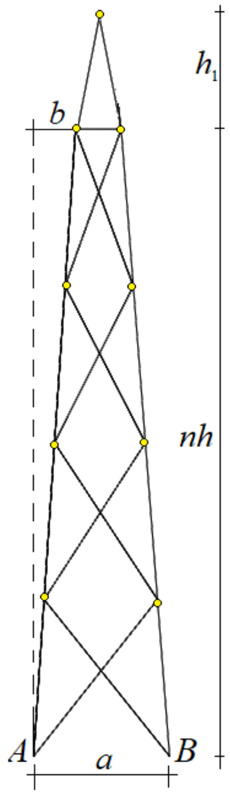

Fig. 2 – The sizes of the truss, n=4

For example, for the coefficient at a^3, the recurrent equation has the form

$$C_{3,n} = 4C_{3,n-1} - 5C_{3,n-2} + 5C_{3,n-4} - 4C_{3,n-5} + C_{3,n-6}. \tag{9}$$

The order of the recurrent equation is two times less than the length required to determine the common term of the sequence. In this problem, it was necessary to calculate 12 trusses with the number of panels from 1 to 12. Note that character conversions in Maple are slow. The time to calculate the natural frequencies of each subsequent truss is approximately twice as long as the previous one. The solution of recurrent equations is given by the *rsolve* operator:

$$C_1 = (35 + 75n - 16(-1)^n) / 36,$$ (10)
$$C_2 = (25n^2 + 4(-1)^n + 13n - 4) / 6,$$
$$C_3 = (3n^4 + 10n^3 + 32n^2 - 2(-1)^n + 21n + 2) / 6,$$
$$C_4 = 2 / 27.$$

Finally, we have an analytical estimate for the lower frequency according to Dunkerley:

$$\omega_D^{-2} = m(C_1a^3 + C_2c^3 + C_3h^3 + C_4d^3) / (a^2EF).$$ (11)

3.2 Numerical solution

To estimate the accuracy of the analytical solution (11), we find numerically the first frequency from the frequency spectrum of the natural oscillations of the structure. The Maple system has a special *Eigenvalues* operator for finding the eigenvalues and vectors of a matrix. Consider a truss with a height of $(n+1)h$ under the condition $h_1 = h$ and with a base of size $a = 6\text{m}$. The cross-sectional area of the rods is the same $F = 10.0\text{sm}^2$. The elastic modulus of the steel is $E = 2.0 \cdot 10^5$ MPa. Figure 3 shows the curves of the dependence of the frequency obtained analytically by the formula (11) and the frequencies obtained numerically from the solution of the problem of the oscillation of the system with the number of degrees of freedom at *r* = 0 and *r* = 0.1 m.

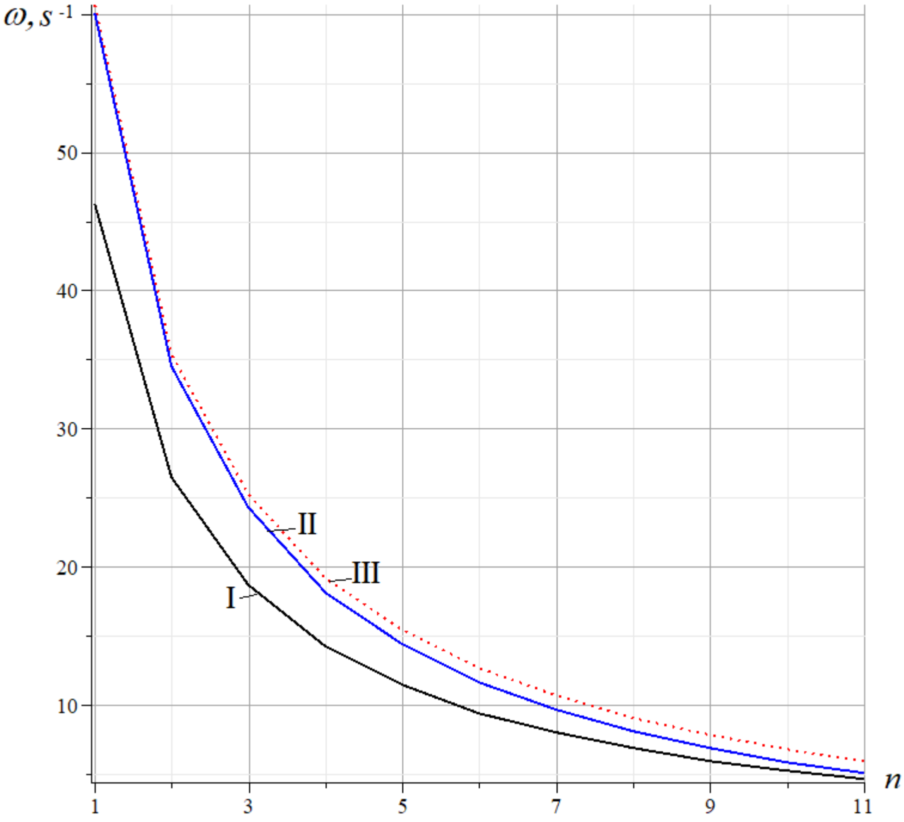

Fig. 3 - The first frequency of vibrations of the truss as a function of the number of panels. I — Frequency ω_D according to Dunkerley, $r = 0$; II — frequency ω_1 obtained numerically, $r = 0.1\text{m}$, III — frequency ω_1, obtained numerically, $r = 0$

The error of the analytical estimate is small. Moreover, this estimate, calculated for a prismatic mast, is quite suitable for the solution obtained numerically for a pyramidal truss with a small slope of the faces. With an increase in the number of panels, the error decreases, which makes the resulting analytical solution particularly attractive for trusses with a large number of panels, where the counting time increases with an increase in the number of panels, and the accuracy decreases.

To clarify the error of the solution, we introduce a relative value of $\varepsilon = (\omega_1 - \omega_D)/\omega_1$. Figure 4 shows the change in this value depending on the number of panels. If we do not count the local maximum, which falls on an unrealistically small number of panels, we can conclude that with an accuracy of about 14%, the analytical estimate gives a satisfactory result. In addition, with an increase in the height of the panels, the error decreases with any number of panels.

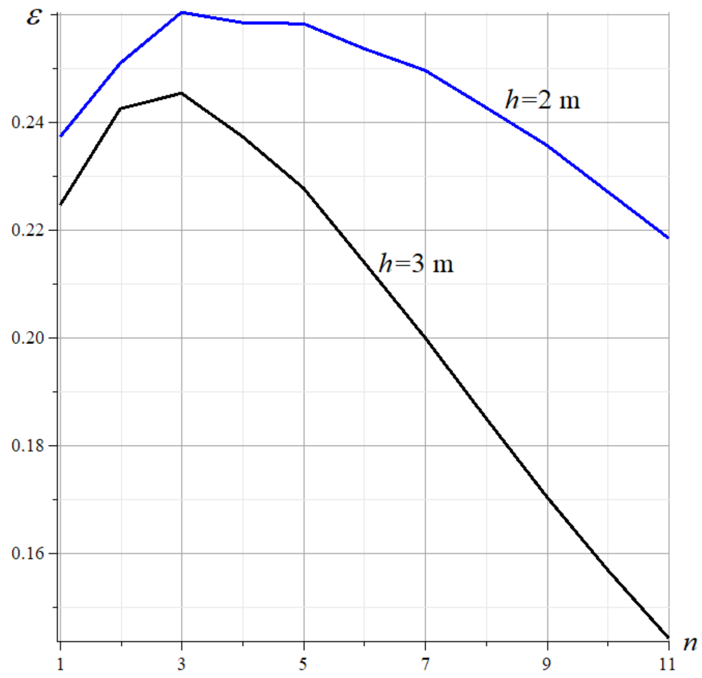

Fig. 4 –The error of the analytical solution depending on the number of panels

4 Conclusions

A mathematical model of a spatial statically determinate truss is constructed. As a result of the analysis of the natural oscillations of the model, the following conclusions can be drawn:

1. The Dunkerley estimate, taking into account an arbitrary number of panels, has a compact appearance and gives acceptable accuracy, especially with a large number of panels.
2. As the height of the truss increases, the accuracy of the analytical solution increases.
3. The solution obtained for the prismatic shape of the structure not only describes well the frequency dependence on the number of panels for the pyramidal shape of the mast but also gives greater accuracy in this case.

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