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Analytical Dependence of Planar Truss Deformations on the Number of Panels

Dai, Qiao [1]

[1] Moscow Power Engineering Institute, Moscow, Russian Federation
Correspondence: email 228441531@qq.com; contact phone +79199909759

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Abstract:

The object of the study is a planar statically determinate truss of the regular type. The truss has a sprengel grid. The load evenly distributed over the nodes of the upper belt is considered. The purpose of the work is to derive an analytical dependence of the deflection of the structure on the number of panels. **Method.** By the method of induction, based on the results of calculations of a series of similar trusses with a consistently increasing number of panels, the desired dependence is derived. All transformations and the solution of the system of linear equations for determining the forces in the rods are performed in the Maple computer mathematics package. The deflection is calculated using the Maxwell-Mohr formula. **Results.** The formula for the dependence of the deflection on the number of panels contains ten coefficients obtained from the solution of recurrent equations. Graphs of the dependence of the deflection and horizontal displacement of the movable support under the action of a uniform load along the upper belt are constructed. The asymptotics of the solutions is found.

1 Introduction

The calculation of the strength, stability and deformations of trusses in the design process is practically carried out by numerical methods in specialized packages based on the finite element method [1]–[3]. In cases where the design contains many discrete elements, such as rods in trusses, numerical methods are subject to such a difficult-to-eliminate disadvantage as the accumulation of rounding errors. Analytical methods for solving problems have become more real with the development of computer mathematics systems (Mathematica [4], Maple [5]–[8], Reduce, Derive, etc.). At the same time, the problem of the universality of formulas for solving engineering problems becomes a very important problem. If the mathematics obtained in any system is intended only for one structure and type of load, then its value is low. One of the tasks in the design is to choose the optimal option for any parameter (or group of parameters). Often, the most optimal truss scheme may not be the initially selected scheme, but a truss with fewer or more panels. Here, it automatically becomes important to enter the order of the structures (the number of panels, for example) into the calculation formula. One of the most common methods for taking into account the order of a regular system in a calculation formula is the method of induction [9]–[12]. This method provides dozens of solutions for girders [13]–[21], arch [22], and lattice trusses. More complex solutions refer to spatial constructions of the regular type [23]. General questions of calculations and analysis of regular systems, as well as the problems of the existence and search for statically determinate structures of this type, are considered in [24], [25]. Solutions to the problems of oscillation of regular trusses are also known [26]–[29]. In this paper, we consider a truss scheme with a complex lattice of the sprengel type. The peculiarity of this scheme is the non-central location of the sprengel node in height, which leads to a significant complication of the solution.

2 Materials and Methods

2.1 Truss scheme

Consider a regular truss with parallel belts and a sprengel grid (Fig. 1). The truss contains *n* identical panels. Each $4a$ – long panel contains four $2a$ – long elements in the upper and lower belt, three $h+b$ – high posts, four struts length $c=\sqrt{a^2+h^2}$, and two struts length $d=\sqrt{a^2+b^2}$. The truss has a total of $\mu=12n+4$ members. The truss is symmetrical.

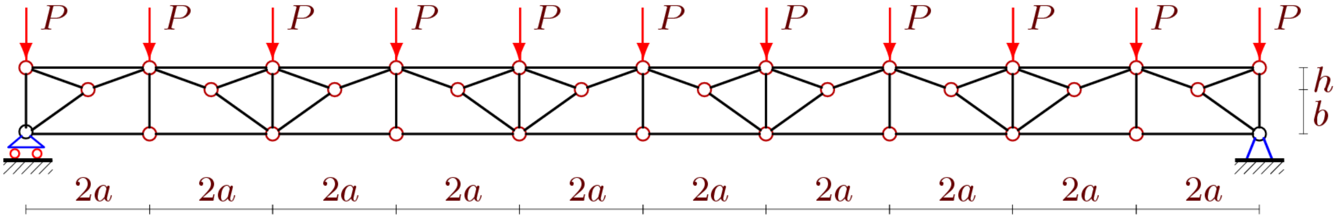

Fig. 1 - Truss, *n*=5, *h*<*b*

To calculate the deflection (vertical displacement of the middle node of the lower belt), we use the Maxwell – Mohr formula. The values of forces included in this formula in a statically definable construction can be found from the system of equations of equilibrium of nodes. To obtain an analytical solution, we will use the Maple computer mathematics system. To enter truss data into the program, the truss nodes are numbered (Figure 2).

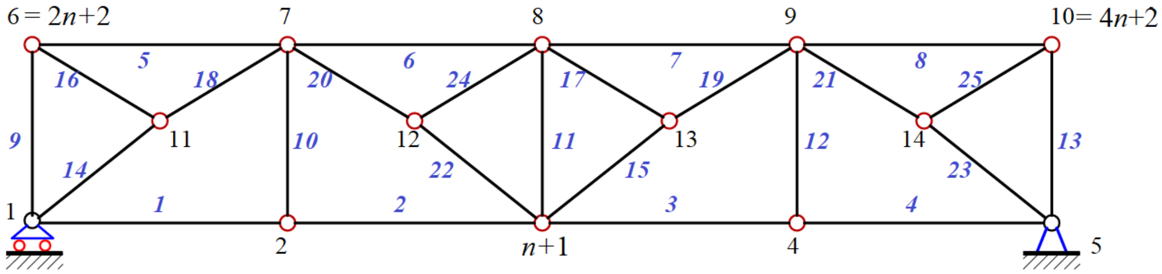

Fig. 2 - Truss, *n*=2 . Numbering of nodes and rods

We will take the origin of the coordinates in the left movable support. The fragment of entering coordinates in the Maple language has the form

```
for i to 2*n+1 do x[i]:=2*(i-1)*a; y[i]:=0;
                  x[i+2*n+1]:=x[i];   y[i+2*n+1]:=h+b; od:
for i to 2*n do   x[i+4*n+2]:=x[i]+a; y[i+4*n+2]:=b; od:
```

The order of connecting the rods is set by special lists numbered by the numbers of the rods, containing the numbers of the ends of the rods.

The rods of the belts are numbered as follows:

```
for i to 2*n do   N[i]:=[i,i+1]; N[i+2*n]:=[i+2*n+1,i+2*n+2];od:
```

Numbering of vertical racks

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for i to 2*n+1 do N[i+4*n]:=[i,i+2*n+1];od:
```

The truss grid is similarly encoded.

2.2 Calculation of forces and deflection

Based on the data on the coordinates and taking into account the structure of the joints of the rods, we calculate the guiding cosines of the forces included in the matrix of the system of equilibrium equations of the nodes.

In the vector of the right part of the system of equations, when determining the forces from the action of the load, external forces are placed — vertical loads on the nodes of the upper belt::

$$B_{2i}=P,\ i=2n+2,..,4n+2\,.$$

When determining the forces $S_{\alpha}^{(1)}$ from the action of the vertical unit force on the node in the middle of the span (at the point *n*+1 of the deflection determination), only one element is distinguished from zero on the right side

$$B_{2i} = 1,\ i = n+1. \qquad (1)$$

The solution of the system, as well as the forces in the rods, is in symbolic form. The deflection is calculated using the Maxwell – Mohr formula

$$\Delta = \sum_{\alpha=1}^{\mu} S_{\alpha}^{(P)} S_{\alpha}^{(1)} l_{\alpha} / (EF), \qquad (2)$$

where $S_{\alpha}^{(P)}$ is the force in the rod with the number α from the action of the load distributed over the nodes, $S_{\alpha}^{(1)}$ is the force from the action of a single vertical force in the node with the number $n+1$ in the middle of the lower belt, l_{α} is the length of the member α. The stiffness of the rods *EF* is taken to be the same. The calculation of individual trusses of different orders shows that the form of the formula for the deflection does not depend on *n*:

$$\Delta = P((C_1 a^3 h^2 + C_2 a^3 b^2 + C_3 b^3 h^2 + C_4 c^3 h^2 + C_5 d^3 h^2 + C_6 c^3 b^2 + \qquad (3)$$
$$+ C_7 h^3 b^2 + C_8 h^4 b + C_9 h^5 + C_{10} a^3 bh) / ((b+h)^2 h^2) EF).$$

Here the coefficients $C_i,\ i = 1,..,10$ depend only on n and are determined by induction

$$C_1 = (20n^4 - 2(3(-1)^n + 1)n^2 - 3(-1)^n + 3) / 6, \qquad (4)$$
$$C_2 = n^2, C_3 = (2(-1)^n - 1 + 2n) / 2,$$
$$C_4 = n^2, C_5 = 2n^2,\ C_6 = n^2,$$
$$C_7 = -(1+2n) / 2,$$
$$C_8 = -(2(-1)^n - 1 + 2n) / 2,$$
$$C_9 = (2n+1) / 2,$$
$$C_{10} = (2(-1)^n n^2 + (-1)^n - 1) / 2.$$

Under the influence of a vertical load, the movable hinge of the left support is displaced. To determine this offset, a horizontal force is applied to the node with the number 1. In the vector of the right part of the system of equilibrium equations for horizontal loads, odd elements are assigned. Instead of (1), we have

$$B_{2i-1} = 1,\ i = 1. \qquad (5)$$

By induction, based on the results of calculations of only eight trusses with a consistently increasing number of panels, we obtain a solution for the shift value

$$\delta_1 = 4Pa^2 n(2n^2 + 1) / (3(h+b)EF). \qquad (6)$$

3 Results and Discussion

The graphical capabilities of the Maple system allow you to give a clear picture of the distribution of forces in the truss rods. For a truss with three panels and with dimensions $a = 3$ m, $h = 2$ m, $b = 4$ m, the force values related to the value *P* are given in Figure 3. The compressed rods are highlighted in blue, and the stretched ones are highlighted in red. The unstrained rods are indicated by thin black segments. The thickness of the lines is approximately proportional to the force modules. The distribution of the compressed rods in the upper belt and the bars in the side panels resembles an arch in shape. This indicates the need for an appropriate distribution of the stiffness of the rods. It is also characteristic that the most compressed rods are not in the middle of the span. On the contrary, the most stretched rods, as expected, are located in the middle of the lower belt.

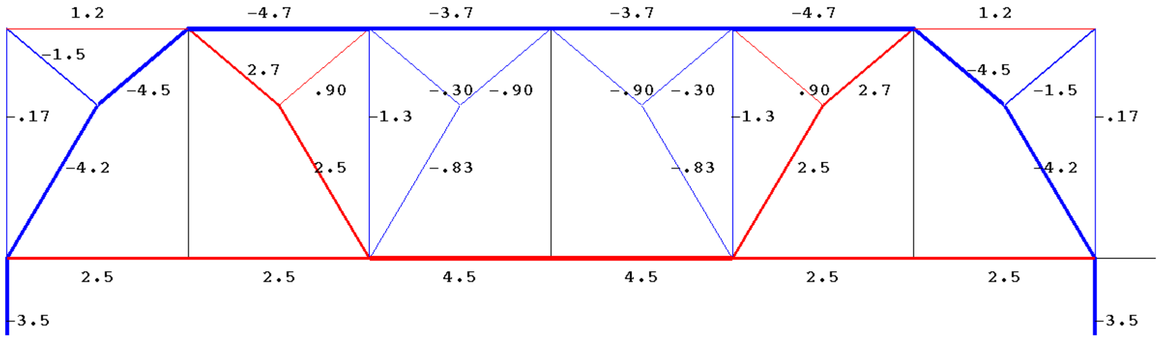

Fig. 3 - Truss, *n=3*. Distribution of forces in the truss rods, $n=3,\ a=3\text{m},\ h=2\text{m},\ b=4\text{m}$

Approximately the same distribution of forces will be at $b<h$ (Fig. 4). The difference is noticeable only in the compressed elements of the upper belt. The maximum compression forces here are 15-30% less.

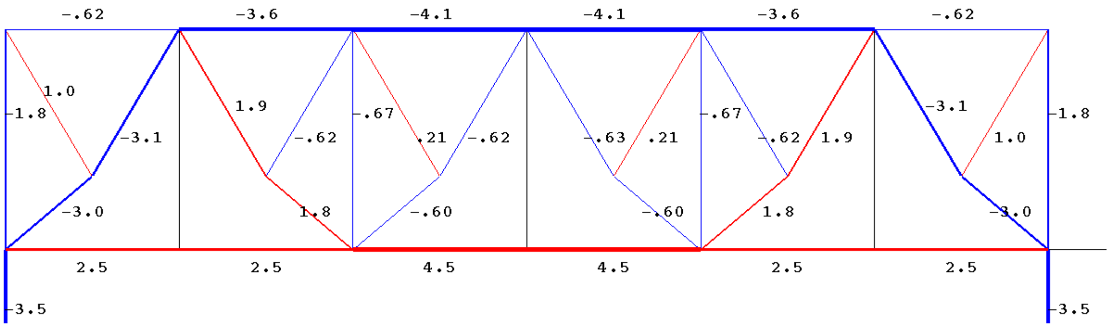

Fig. 4 - Truss, *n=3*. Distribution of forces in the truss rods, $n=3,\ a=3\text{m},\ h=4\text{m},\ b=2\text{m}$

To illustrate the found dependence of the deflection on the number of panels, we construct the corresponding graphs for the length $L=2na$ truss with the total load $P_0=(2n+1)P$. We introduce a relative dimensionless deflection $\Delta'=EF\Delta/(P_0L)$. Figure 3 shows the curves for this value, constructed according to formula (4). For all curves, the height of the truss is taken to be 10 m. Curve 1, corresponding to $b=2\text{m},\ h=8\text{m}$, shows that such a truss (*b*<*h*) is the most rigid. As the number of panels increases, curves 1 and 2 converge, and this convergence depends on the parity of *n*. It is also obvious that the change in the deflection value is different depending on the ratio of b and h. At *b*<*h*, the deflection increases non-monotonically, and at *h* < *b*, the deflection decreases to a certain point also non-monotonically. The graph at *n*>12 (Figure 4), which is a continuation of the graph in Figure 3, shows that all curves have minima. Moreover, it turns out that the found dependence in the accepted formulation (with an increase in the number of panels, the lengths of the panels decrease and the loads on individual nodes decrease) has an oblique asymptote. You can calculate the angle of inclination from finding the limit:

$$\lim_{n\to\infty}\Delta'/n=(2b^2-bh+h^2)/(2L(b+h)).$$

The solution (6) for the relative shift $\delta'=EF\delta/(P_0L)$ also has an asymptote, but it is a horizontal asymptote:

$$\lim_{n\to\infty}\delta'/n=L/(3(b+h)).$$

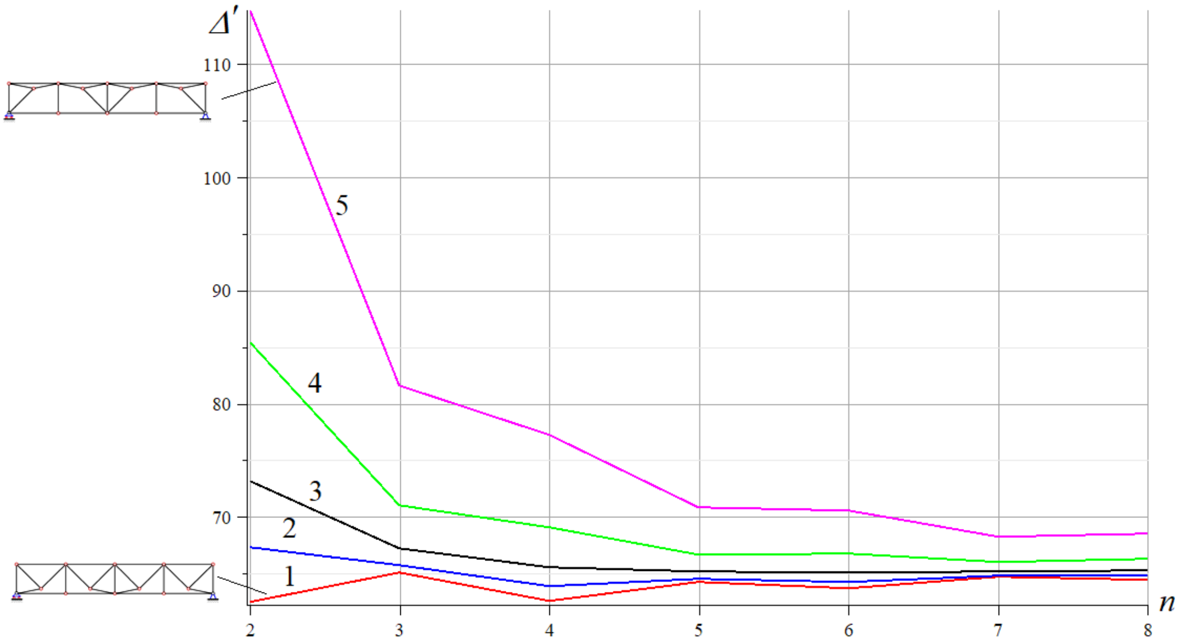

Fig. 3 - Dependence of the relative deflection on the number of panels, $2 \le n \le 8$

1 — $b = 2\,\text{m}; 2$ — $b = 4\,\text{m}; 3$ — $b = 5\,\text{m}; 4$ — $b = 6\,\text{m}; 5$ — $b = 7\,\text{m}$; $L = 180$ m; $h + b = 10$ m

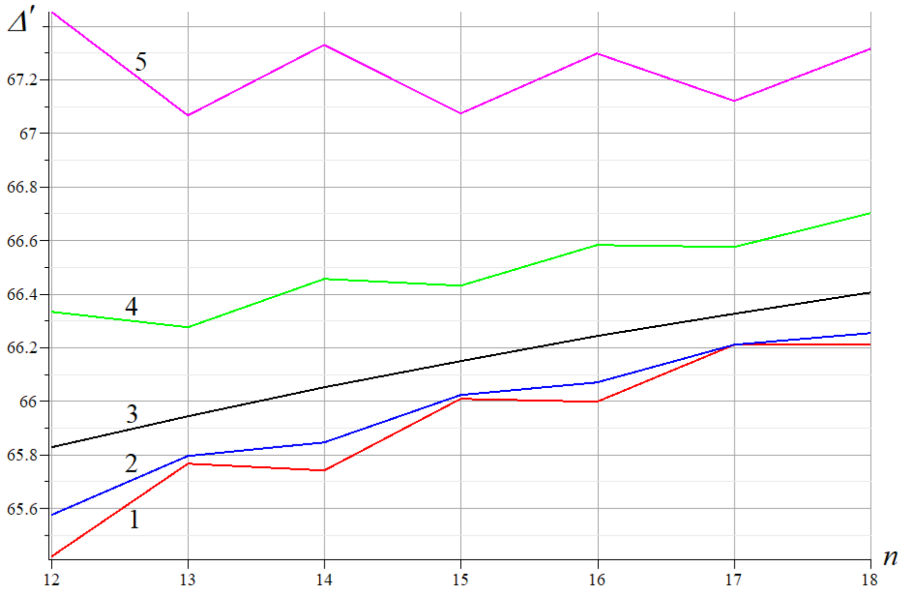

Fig. 4 - Dependence of the relative deflection on the number of panels, $12 \le n \le 18$

Figure 5 shows the dependence of the deflection on the value b at a constant total height of the truss $h + b = 10$ m. The characteristic of these dependencies is the presence of a fairly clearly defined minimum and the self-intersection of the curves at *n* =9 and *n* =10. The intersection point means that trusses of the same length, but with different panel lengths, have the same deflections.

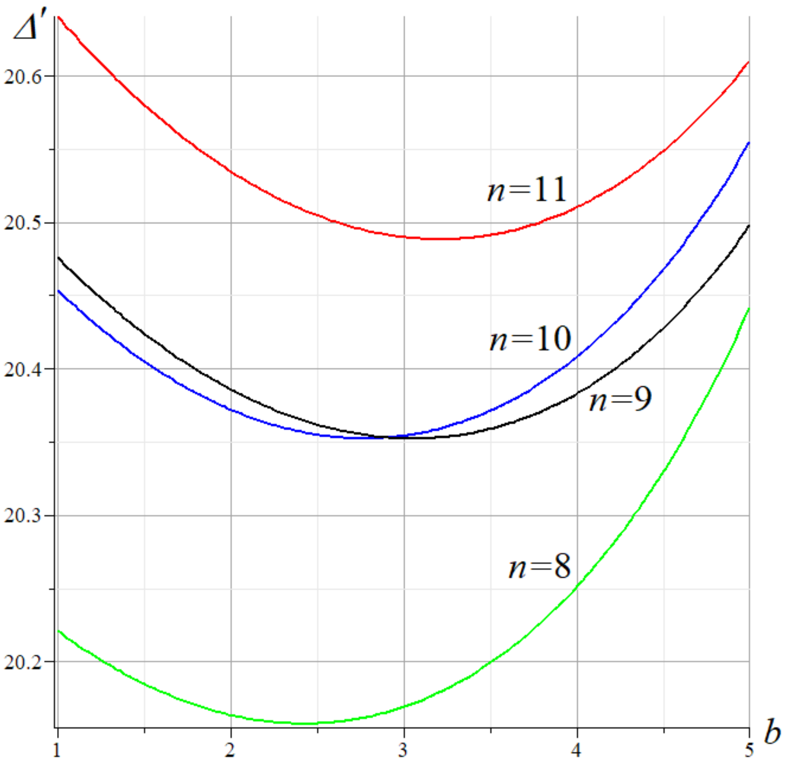

Fig. 5 - Dependence of the relative deflection on the height *b*. $L = 100$ m; $h + b = 10$ m

4 Conclusions

The main results of the work are as follows.

1. A scheme of a statically definable planar regular truss with a sprengel-type lattice is proposed.

2. The dependence for the deflection of the truss and the horizontal displacement of the movable support under the action of a uniformly distributed load is obtained.

3. The features of the distribution of forces on the truss rods for different ratios of the dimensions of the grid elements in the height of the structure are shown.

4. The existence of linear asymptotic solutions for the deflection and shift of the support by the number of panels is shown.

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